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Classical and Quantum Kalman Filter: an application to an Under-actuated Nonlinear System as a Gantry crane

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Abstract: In this work, a novel application of a Kalman filter to an under-actuated nonlinear system is proposed. The theory of the Kalman filter is defined considering both the classical case and the quantum filter. A description of an architecture for a Quantum motion controller is given, highlighting the most important features. In the classical case, an application of a Kalman filter to an under-actuated nonlinear system is developed. At the end, an implementation of a motion profile includinga Kalman filter in the motion equations relatively to the payload sway in a gantry crane is described.
Keywords: Kalman filter, Quantum motion control, Under-actuated Nonlinear system, gantry cranes.

1. Introduction

An important line of the recent research in the field of quantum applications is the Open quantum system technique. Particularly, in this scope, the field of the quantum Motion control was recently developedby Caporali[1].Motion control is a subfield of automation including the systems involved in the control of moving parts of automatic machines. The main components typically include a motion controller, an energy amplifier, and one or more prime movers or actuators. Motion control may be either an open-loop or a closed-loop. We focalize our attention only on closed-loop systems,where the measurement of the considered physical dimension (position, velocity, etc.) is converted to a signal that is sent back to the controller, and the controller compensates for any error. The position or velocity of the automatic machine is controlled using some kind of devices such as linear actuators, or electric motors, generally inverters or servo-drives. Typical examplesof inverters control in automation can be seen either in the recent papers [2], [3]. The quantum motion control developed in the work [1]was a system for defining a motion control system, using quantum-entangled photons to transmitwithout jitter the data to the axes and using a quantum inference Unit to generate a set of nonlinear control gains. This Unit works using a quantum PID controller[4], which feeds back classical information arising from measurement. In this environment, we develop a quantum Kalman filter for non-linear systems.If we wish to take the measured output of a quantum systemand perform classical manipulations such as information processing and feedback, really we have a hybrid classical-quantum description of the system. The adopted approach is based on the notion of a controlled quantum stochastic evolution described by Bouten and van Handel [5]–[7]. It brings to the distinction between the input and the output pictures which is already implicit in their works.

In this paper, we define a Kalman filter to describe the motion equations,developing the theory in the classical case and in the quantum case. Particularly, in the classical case,we developan application for an Under-actuated Nonlinear System. In practice, many control problems involve the “under-actuated” behavior of mechanical systems. In under-actuated systems, the number of equipped actuators is less than that of the controlled variables. That is, actuators do not directly control several degrees of freedom. As an illustration example, nonlinear feedback control of an overhead crane is presented to investigate the proposed theory.

This paper is organized as follows. In Section 2, we describe the architecture of a Quantum motion control system,detailing the different elements. In Section 3, a detailed description of a classical and quantum Kalman filter is given, emphasizing the difference and the similarities. In Section 4, we will consider an application of a classical non-linear Kalman filter to a mechanical under-actuated system as a gantry crane. At the end, in Section 5concludingremarks and possible developments are defined.

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2. Quantum Motion Control

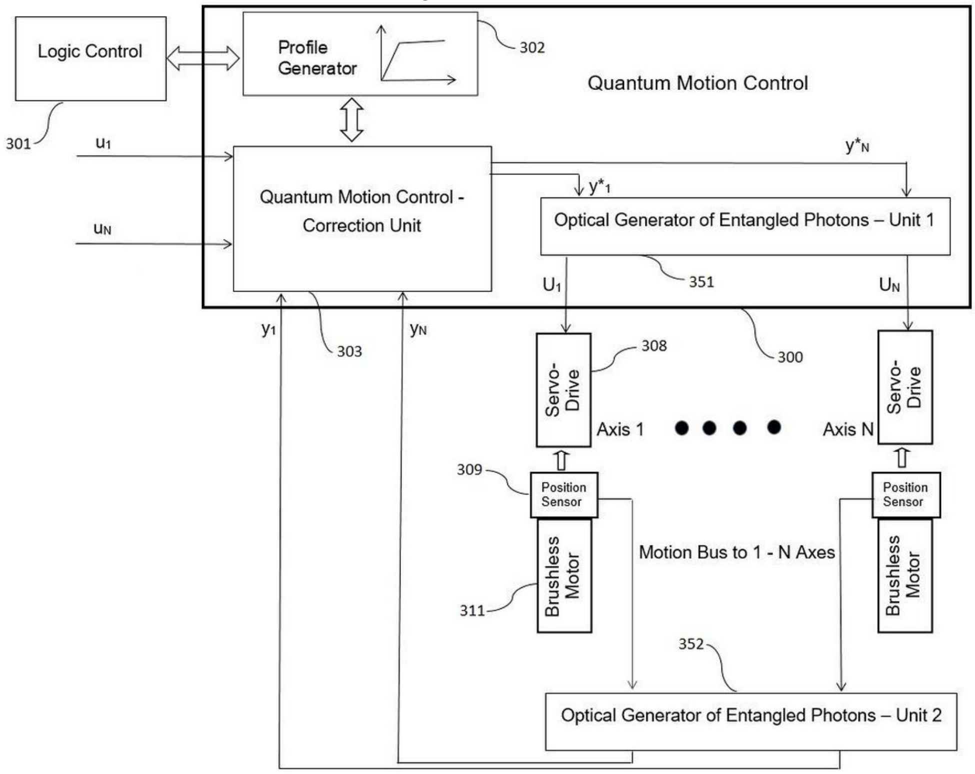

Figure 1: Architecture of a Quantum Motion control

The architecture of a quantum Motion control was recently developed (Caporali[1]).Referring to Figure 1, there is depicted, in a detailed way, an architecture of a Quantum Motion control.The motion bus is a synchronous bus, which is so-called because all the activities on that bus are synchronized using a clock. Therefore, as far as the deterministic part of the motion bus is concerned, it is fundamental that there is a synchronism as exact as possible between the communications that the master makes with the N slave axes. The great advantage obtained with a quantum motion control compared to the existing motion controls is given by the fact that two optical generator units of entangled photons are introduced. That is made in order to perform a perfect synchronism between the N target positions transmitted from the master (virtual) to the N slaves, as well as in order to perform a perfect synchronism between the actual N positions transmitted by the position sensors to the master controller.The central device 300 of the quantum motion control includes means for elaborating the necessary information in order to control the position of the N axes in movement. These means consist of a correction unit 303 included in the quantum motion control device. Furthermore, these means consist also of a profile generator 302 for generating a motion profile, having the task of generating a reference profile for the actuator movement. Again, the central device 300 of the quantum motion control includes also the means to generate optical information consisting of entangled photons. These means can consist, as in the realized example, of a first device 351 having the task of being an optical generator of entangled photons.

In turn, the profile generator 302 includes means to exchange information with the logic control device 301.

Concretely, the logic control device 301, as defined in this invention, can be a programmable logic controller.

This device can be physically separated from the central device 300 of the quantum motion control or it can be included in the central device 300. It follows that the information carried by the entangled photons reaches the servo-drives 308 and, then, the phenomenon of decoherence occurs. This, however, does not affect the information already conveyed and delivered to individual drivers via the optical path. The position sensors 309 of the *N*axes 1, 2,…,*N* must send to the correction unit 303 the information given by the actual position of the corresponding axis. The position sensor is connected to the brushless motor 311 or, in any case, to the electric motor that produces the necessary energy to move the mechanical part of the axis. The information given by the actual position of the single-axis is sent, in a preliminary step, to a second device 352 having the task of being an optical generator of entangled photons.

3. Classical and Quantum Filtering

In a general sense, the goal of filtering theory is tomake an optimal estimate of the state of a system.The systemmay have noisy dynamics, and also is subject to observation noise.

3.1 Non-linear continuous Kalman filter

The Kalman filter is a recursive estimation scheme, requiring minimal storage of information, valid for Gaussian models. A straightforward generalization of the Kalman filter to continuous time is known as the Kalman-Bucy filter. Wewill derive it based on the paper by Kushner [8].We consider a continuous-time nonlinear dynamical extension of the estimation problem where the state undergoes a diffusion. Suppose that we have a system described by a process $\mathrm{v}(X_t)$, $X_t$being the systemobservable. We obtain information by observing a related process $\mathrm{h}(X_t)$.The continuous-time equations are takenas:

$dX_t = v(X_t)dt + \sigma(X_t)dW_t$ (Stochastic dynamics); (1)

$dY_t = \mathrm{h}(X_t)dt + \eta dV_t$ (Noisy observations); (2)

Here we assume that the dynamical noise W and the observational noise V areindependent Wiener processes.Considering a function $\mathrm{f} \equiv \mathrm{f}\,(X_t)$, from the derivation using the Itō calculus, we obtain:

$df_t = \mathcal{L}f_t dt + \sigma_t f_t^{'} dW_t.$ (3)

The generator $\mathcal{L}$of the diffusion process, considering the Itô-calculus, is given by: $\mathcal{L}f = vf^{'} + \frac{1}{2}\sigma^2 f^{''}$. Then, considering the conditional expectation $\widehat{f_t} = \mathrm{E}[f(X_t)|Y_t]$, it will satisfy the Kushner-Stratonovich equation:

$d\widehat{\mathrm{f}_t} = \widehat{\mathcal{L}\mathrm{f}_t}\mathrm{dt} + \{\widehat{\mathrm{fh}_t} - \widehat{\mathrm{f}_t}\widehat{\mathrm{h}_t}\}dI_t,$ (4)

where the innovations process is given by:

$dI_t = dY_t - \widehat{\mathrm{h}_t}dt.$ (5)

The innovations process is a Wiener process.

3.2 Quantum Kalman filter

With regard to the Quantum filter and feedback controls, we refer to a rich bibliography:relatively to the quantum stochastic equations we refer to [8]-[13], to the quantum filter and to the quantum input-output fed-back we refer to [5]-[7] and to [14]-[17].

Here we considerthe semi-Markov processes for studying the problems of quantum observation and feedback control outlining their solutions. In typical quantum mechanics, whichtreats only closed Hamiltonian quantum dynamics of unobserved microsystems, there is no observation problem since the measurement. In an open system with feedback control, we need to take into account that processes Z (input process) and Y (output process) are incompatible. Therefore, it is better to view them as referring to two different pictures, which we may call the input picture and the output picture. In the input picture, we describe the world through the controlling process Z. More specifically, we follow the Hudson-Parthasarathy theory where the SLH-coefficients are not fixed system operators, but generally adapted processes commuting with Z. In the output picture, we describe the world through the measured process Y. Just as with the Schrödinger and Heisenberg pictures, it is possible to switch from one to the other.

In the Heisenberg picture, let be$|\psi_0\rangle$the initial state, then if we set $\langle\psi_0|\hat{X}_t|\psi_0\rangle = \langle\psi_t|X|\psi_t\rangle$, we obtain

$d\hat{X}_t = \widehat{\mathcal{L}X}_t + \{\widehat{XL}_t - \hat{X}_t\hat{L}_t\}dI(t) + \{\widehat{L^*X}_t - \widehat{L^*}_t\hat{X}_t\}dI(t)$,where: (6)

$\mathcal{L}X = \frac{1}{2}\sum_i L_i^*[X, L_i] + \frac{1}{2}[L_i^*, X]L_i - i[X, H]$is the Linbladian generator. (7)

The square parenthesis is the commutator $[X, H] = XH - HX$,

and the Innovations process is given by:

$dI_t = dY_t - (\widehat{L_t + L_t^*})dt.$ (8)

I(t) is a Wiener process, and its variation dI(t) is the difference between what we observe dY(t) and what we expect to obtain, that is$\langle\psi_t|L + L^*|\psi_t\rangle$.

Without loss of generality, we consider a single input process. We will take into account a von Neumann commutative algebra$\xi_t = vN\{Z(s): 0 \le s \le t\}$,and in this context we consider an adapted process F(t), in which variation is given by:

$dF(t) = \left\{LdZ(t) - \left(\frac{1}{2}L^*L + iH\right)dt\right\}F(t)$,F(0)=I. (9)

We will consider$\zeta_t(X) = \mathbb{E}[F(t)^*XF(t)|\xi_t]$, that is the input picture conditional expectation with respect to the input picture process Z.$\sigma_t(X)$will be the output picture conditional expectation with respect to the output picture observation Y. $\sigma_t(X)$will be defined as:

$\sigma_t \triangleq U_t^*\mathbb{E}[F(t)^*XF(t)|\xi_t]U_t.$ (10)

U(t) is the adapted unitary process describing the evolution of the system in the Interaction picture.

The general form of the constant operator-coefficientU(t) is given by the quantum stochastic differential equation:

$$dU(t) = \left\{-\left(\frac{1}{2}L_k^* L_k + iH\right)dt + \sum_j L_j\, dB_j^*(t) - \sum_{j,k} L_j^* S_{jk}\, dB_k(t) + \sum_{j,k}\left(S_{jk} - \delta_{jk}\right)d\Lambda_{jk}(t)\right\}U(t) \quad (11)$$

where the triple parameters (S, L, H) are termed the Hudson-Parthasarathy parameters or, informally, the S-L-H coefficients. For more details on the S-L-Htheorysee[12, [13], [14], [16].

Setting $\pi_t(X) = \sigma_t(X)/\sigma_t(I)$and $\varpi_t(X) = \zeta_t(X)/\zeta_t(I)$, we will have the output and input filter normalized $\pi_t(X), \varpi_t(X)$ and not $\sigma_t(X), \zeta_t(X)$ that are not normalized. Exactly:

$$d\zeta_t(X) = \zeta_t(\mathcal{L}X)dt + \{\zeta_t(XL + L^*X)\}dZ(t) \quad (12)$$

$$d\varpi_t(X) = \varpi_t(\mathcal{L}X)dt + \{\varpi_t(XL + L^*X) - \varpi_t(X)\varpi_t(L + L^*\}[dZ(t) - \varpi_t(L + L^*)dt] \quad (13)$$

$$d\sigma_t(X) = \sigma_t(\mathcal{L}X)dt + \{\sigma_t(XL + L^*X)\}dY(t) \quad (14)$$

$$d\pi_t(X) = \pi_t(\mathcal{L}X)dt + \{\pi_t(XL + L^*X) - \pi_t(X)\pi_t(L + L^*\}[dY(t) - \pi_t(L + L^*)dt] \quad (15)$$

4. Classical application of a Kalman filter to an under-actuated Non-linear system

In order to implement an application of a non-linear Kalman filter, we will consider a mechanical under-actuated system. We model a mechanical system by a set of nonlinear differential equations in which the mathematical model is divided into two subsystems: one for actuated outputs and the other for un-actuated outputs.In practice, many control problems involve the "under-actuated" behavior of mechanicalsystems [18]. In under-actuated systems, the number of equipped actuators is less than that of the controlled variables. That is, actuators do not directly control several degrees of freedom.

For dynamical systems, a mathematical model isconstructed based on mechanics principles, such as Newton's law, Lagrange equation, Lagrange multiplier method, Euler-Lagrange methodology, and so on. In mechanical systems with multiple degrees of freedom, system dynamics will comprise a set of second-order differential equations in terms of displacements $\mathbf{q}$, velocities$\dot{\mathbf{q}}$, and time **t**.According to [19], a mechanical system that can be described mathematically by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{u} \quad (16)$$

is regarded as an under-actuated system if the rank of matrix **B(q)** is less than the dimension of vector **q**, that is,rank(B(q))<dim(q).

In general, the physical behavior of a MIMO (multiple input, multiple output) mechanical system is governed by a set ofdifferential equations of motion. Consider an under-actuated system with *n* degrees of freedom driven by *m* actuators ($m<n$). The mathematical model, which is composed of *n* ordinary differential equations, is simplified in matrix form as follows:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{F} \quad (17)$$

Where$\mathbf{q} \in R^n$ is the vector of the generalized coordinates, and $\mathbf{F} \in R^n$denotes the vector of the control inputs. Given that the system has more control signals than actuators, **F** has only *m* nonzero components; $\mathbf{M}(\mathbf{q})$is the symmetric mass matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$is the Coriolis and centrifugal matrix and $\mathbf{G}(\mathbf{q})$is the Gravity vector.

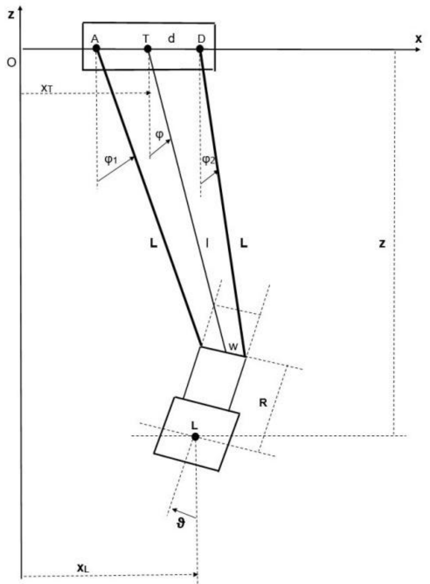

Figure 2:Geometric description of a Gantry Crane

As an application of a mechanical under-actuated system, we consider the case of a gantry crane. In this case, as reported in [3], andfocusing our interest relative to the Lagrangian coordinate φ(the sway angle), in the first approximation, we can obtain (see [3]):

$$\{l\ddot{\varphi} + \ddot{x}\cos\varphi\} - \{(\dot{l} - k_f)\dot{\varphi}\} + g\sin\varphi = 0, \quad (18)$$

Wherel is the central cable length, $x=x_L$ the horizontal position of the crane, g the gravity, $k_f$the friction coefficient. Compared with eq. (17), we see the terms $\ddot{\mathbf{q}} \equiv \{l\ddot{\varphi} + \ddot{x}\cos\varphi\}$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \equiv (\dot{l} - k_f)$, $\mathbf{G}(\mathbf{q}) \equiv \{g\sin\varphi\}$.

With reference to [20], we consider the Brownian motions $W_1(t)$ and $W_2(t)$ and the constants σ_{11}, σ_{12}, σ_{21}, σ_{22}. Then, starting from eq. (17), we set:

$$\dot{\varphi}_1 = \varphi_2, \quad (19)$$
$$\dot{\varphi}_2 = 1/l\{-\ddot{x}\cos\varphi_1 + (\dot{l} - k_f)\varphi_2 - g\sin\varphi_1\}.$$

Therefore, according to [20], we can form the following two Itô processes:

$$d\varphi_1(t) = \varphi_1 dt + \sigma_{11} dW_1 + \sigma_{12} dW_2, \quad (20)$$
$$d\varphi_2(t) = \varphi_2 dt + \sigma_{21} dW_1 + \sigma_{22} dW_2,$$

Following eq. (1)-(3), we can obtain:

$$df_1(t) = \varphi_1 dt + \sigma_{11} dW_1 + \sigma_{12} dW_2, \quad (21)$$
$$df_2(t) = 1/l\{-\ddot{x}\cos\varphi_1 + (\dot{l} - k_f)\varphi_2 - g\sin\varphi_1\}dt + 1/2(\sigma_{21} + \sigma_{22})dt + \sigma_{21} dW_1 + \sigma_{22} dW_2.$$

Equation (21), together with eq. (2)which is referred to the related measurement process $h(X_t)$, defines the system for developing the solution that takes into account the classical Kalman filter.

5. Implementation of a motion profile with a Kalman filtering for an under-actuated system

The whole system of governing equations and the corresponding iterative process was simulated in CodesysV3.5 SP7. That is because Codesys, written in Structured Language (SL), actually is the most common way to realize function blocks in an industrial motion control environment.

The cyclic task, in which the function block of the used Plc was realized, had a time of updating equal to 30ms.The function blocks (FB's) used were three: the first FB computes the speed profileof the crane necessary to obtain the anti-sway functionality and the corresponding actual sway angleφ_1.

The second FB is used to compute the actual length *l* of the cable and the corresponding vertical speed using the data coming from an external unit (i.e., encoders) connected to the motor of the corresponding movement.

The third FB computes the statistics correction, generating the Gaussian profile relative to the Wiener processes dW_1 and dW_2.

The speed reference is the target value to which the speed must arrive, controlledeither bythe crane operator or by the automatic control. Usually, it is defined in Hz, as a consequence of the way the electric motor velocities are defined. At the speed in Hz on the fast shaft (that is on the motor) a velocity corresponds on the slow shaft (that is on the wheels moving on either the rails of the trolley or the shaft controlling the slewing motion of the jib), depending on the reduction gearing from the motor to the wheel. Typically, the max speed of a crane can be from 0.2 m/s to also more than 2 m/s. The ramp set is the value of the time that would be required for the linear motor ramp to reach the speed reference.

Based on the speed reference and on the ramp set, the estimator module in the Plc computes the real speed profile in order to have the anti-sway effect. The generated speed profile is longer than the linear ramp set.

The cable length has a very important influence on the speed profile, because the greater the cable length, the greater the time of the speed profile is.

In the practice of crane control, it is fundamental to reduce the time of the speed profile together with getting the anti-sway effectin order to minimize the time of the wanted movement.

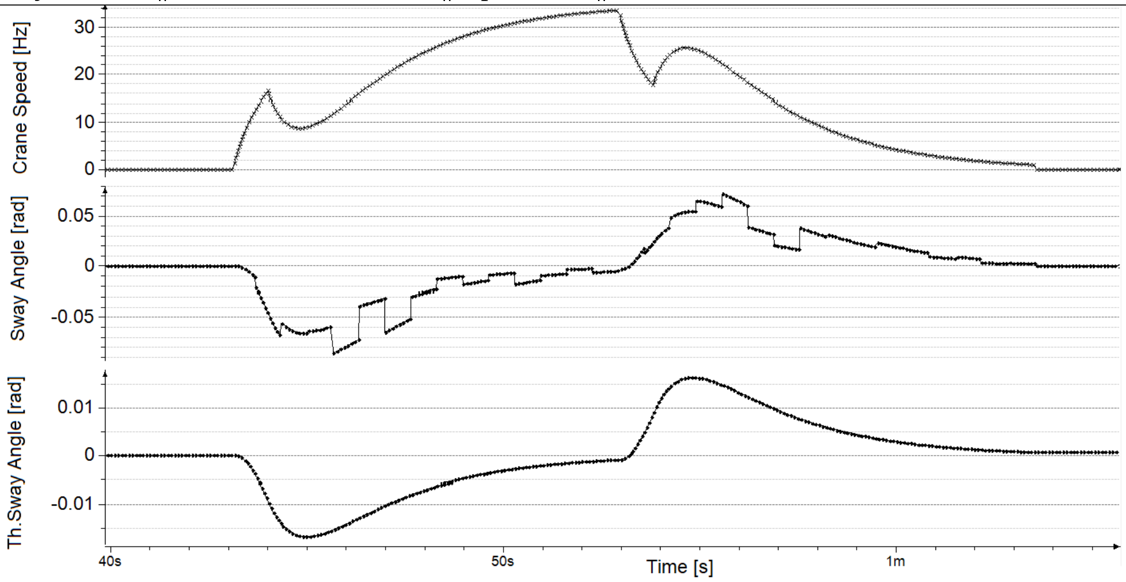

Figure 3: Speed profile and Sway angles with high values of the Kalman filter correction.

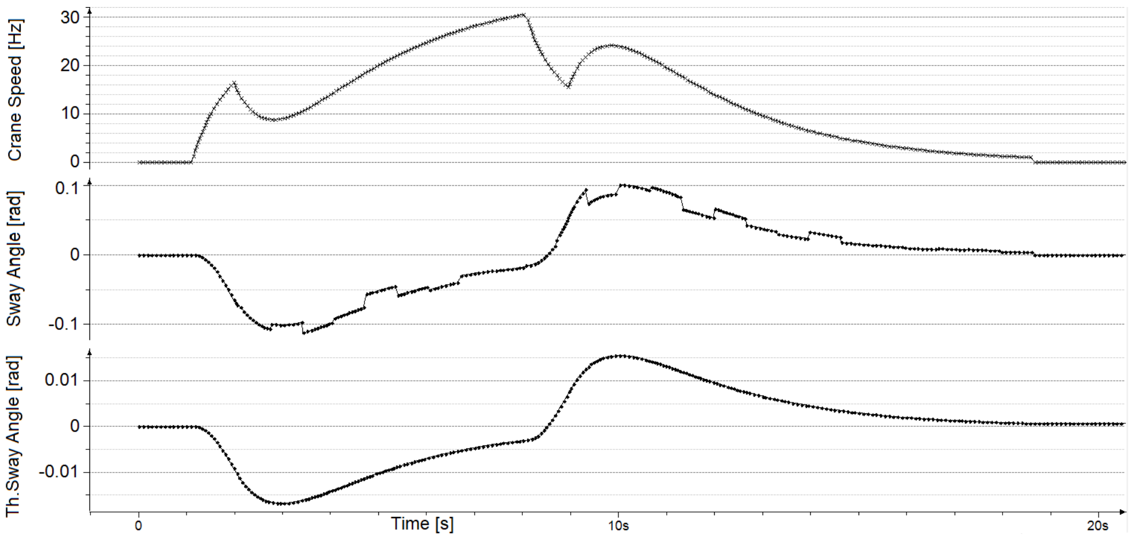

Figure 4: Speed profile and Sway angles with low values of the Kalman filter correction.

In Figure 3, we can see the velocity profile for the crane movement, the corresponding profile of the sway anglewith the correction due to the observation process and the statistics correction due to the non-linear Kalman filter and the corresponding profile of the theoretical sway anglewithout the corrections.The specific values of the inputs received from the estimator module are: speed reference = 35Hz (corresponding to 1.0m/s), ramp set = 1.5s, cable length = 11.5m, $\eta = 2$(η is the parameter of the Kalman filter in eq. 2).

In Figure 4, the same variables are described corresponding to different values of the parameters relative to the observation process and to the statistics correction. That is in correspondence to the same defined movement, described in the figures.The specific values of the inputs received from the estimator module are: speed reference = 35Hz (corresponding to 1.0m/s), ramp set = 1.5s, cable length = 11.5m, $\eta = 5$.

We can see, observing them as a whole in Figures 3 and 4, that at the end of the crane movement, the sway angles are canceled. Therefore, the desired anti-swaying effect is obtained. That is obtainedby optimizing the profile for the crane movement.

The velocity reference profiles of the crane shown in the present work agree well with the velocity reference profiles found in the previous work of the author [3].

We also note the profile of the real Sway angle compared with the theoretical profile of the same Sway angle. It appears evident, as a consequence of the observation process and of the statistics correction due to the non-linear Kalman filter, that the real profile of the sway angle is not continuous.We see that the most important corrections on the values of the sway angle occur when high values of the Kalman filter correction are set, as is logical to expect.

6. Conclusions and future work

In this paper, we defined a Kalman filter to describe the motion equations, developing the theory in the classical case and in the Quantum case. In the classical case, we developed an application for an Under-actuated Nonlinear System. Besides, a description of a Quantum motion controller is given.

The tests showed substantial differences between the real and the theoretical profiles as a consequence of the observation process and of the statistics correction due to the non-linear Kalman filter.

These results and the definition of the Quantum Motion Controller encourage us to expand the Under-actuated Nonlinear Systemapplication to a quantum system using a Quantum Kalman filter.

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