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Bilinear Delay

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Abstract: In this paper, the most important characteristics of the bilinear delay for differential expressions of the Sturm Liouville type of Sturm liquids with variable delay are analyzed. Because of the width of the problem, certain conditions for the troparameter family from the set R.
Key words: differential operator, bilinear delay, troparameter family

Differential expression type

$$-y''(x) + q(x)y(\gamma(x)), \quad x \in [0,\pi] (1)$$

with functions$y,\ y'',\ q,\ \gamma$ belong to one of the functional operators and applies $\gamma(x) < x, x \in [0,\pi]$generates a differential operator Sturm to Liouville type with delays in the segment$[0,\pi]$.Function y is function of delay in form[5].

$$\gamma(x) = x - \frac{ax+b}{x+c}\ , (a,b,c) \in R^3 (2)$$

we set the conditions

$1'$ $\gamma \in C^{\infty}[0,\pi]$
$2'$ $\gamma(0) < 0$
$3'$ γ strictly growing to $[0,\pi]$
$4'$ $\exists\, \xi_1 \in (0,\pi)\colon \gamma(\xi_1) \in (0,\pi)$
Condition *1'* is satisfied for$a^2 + b^2 > 0$ and $c \in (-\infty,-\pi) \cup (0,+\infty)$.
Condition *2'* means-$\frac{b}{c} < 0$ that is $bc > 0$

Thus, the troparametric family (3) satisfies conditions 1 'and 2' at the set
$A = \{(a,b,c) \in R^3 \colon a \in R\backslash\{0\},\ \ b \in R\backslash\{0\},\ \ c \in (-\infty,-\pi) \cup (0,+\infty),\ \ bc > 0, ac < b\} (2')$

Function γ strictly grows if it is

$$\gamma'(x) = \frac{x^2 + 2cx + (c^2 - ac + b)}{(x+1)^2}$$

Positive to $[0,\pi]$.
Therefore we are studying the condition

$$x^2 + 2cx + (c^2 - ac + b) > 0, x \in [0,\pi].$$

Since the nuances of trinity date with

$$x_{1,2} = -c \pm \sqrt{b(a-1)}$$

We differentiate between cases
$(3',1)\ \ b(a-1) \le 0$
Under this condition the trine is strictly negligent andfunctions γstrictly growing.
Cause of $b \neq 0$we have :

$$b < 0 \quad \text{and} \quad a \ge 1 \ \text{ or}$$
$$b > 0 \text{ and } \ 0 \neq a \le 1.$$

$(3',2)\ \ b(a-1) > 0.$
It should be ensured that a higher zero trinity is non-positive, leading to arelation

$$-c + \sqrt{b(a-1)}\ \le 0 \ \text{ and } \sqrt{b(a-1)} < c.$$

This means he is $c > 0$ sowhy $bc > 0$ means $b > 0$ and $a > 1$. So by constructing a three

$(a,b,c) \in R^3$with the conjunction of conditions $(3',1)$and$(3',2)$we come to the set B which defines a troparametric family of function γ satisfying the conditions 1', 2' and 3' satisfied at the gathering

$$B = \Big\{(a,b,c) \in R^3 : \Big((a,b,c) \in [1,\infty) \times R^- \times (-\infty,-\pi) \vee \big((a,b,c) \in R\backslash\{0\} \times R^- \times R^+\big)\Big) \\ \vee \big((a,b,c) \in (0,1) \times R^- \times (-\infty,-\pi) : ac > b\big)\Big\} \quad (3)$$

Thus we proved the result given in the form of the next theorem

Theorem 1 To function γ set with (2) in the segment $[0,\pi]$ was infinitely variable,strictly growing, and zero negative, it is enough that the vector (a,b,c)belongs to the set $A \cup B$ where A is defined by (2 ') and B by (3). In the further discussion, we need a vector request $(a,b,c) \in B$to meet the condition 4' that is :

$$\exists\, \zeta_1 \in (0,\pi) : \gamma(\xi_1) = 0,$$

respectively

$$\xi_1 - \frac{a\xi_1 + b}{\xi_1 + c} = 0$$

$$(\xi_1)^2 + (c-a)\xi_1 - b = 0$$

$$(\xi_1)_{1,2} = \frac{1}{2}\Big[a - c + \sqrt{(a-c)^2 + 4b}\Big].$$

The first requirement is to apply $(\xi_1)_{1,2} \in R$that is

$$(a-c)^2 + 4b \geq 0, \quad b \in \left[-\frac{(a-c)^2}{4}, 0\right) \cup (0,\infty)$$

If it is $b = -\frac{(a-c)^2}{4}$, $\xi_1 = \frac{a-c}{2} > 0 \Rightarrow a > c$. Cause of $bc > 0$, it must be $c < 0$.
For $b \in \left(-\frac{(a-c)^2}{4}, 0\right)$ must be zero $(\xi_1)_2$ be negative and the requirement to get zero $(\xi_1)_1$ positive leads to conditions

$$\sqrt{(a-c)^2 + 4b} > c - a \quad (*)$$

The condition (*) is satisfied for all points $(a,b,c) \in B$for which it is$c < a$.However, if it is$c \geq a$relation (*) leads to the conditions$b \geq 0$which is contradictory to the condition
$b \in \left(-\frac{(a-c)^2}{4}, 0\right)$. If so $b > 0$, must also be $c > 0$ so be it $\xi_1 \in R$ for all vectors from $A \cup B$.
Then

$$(a,b,c) \in R\backslash\{0\} \times R^+ \times R^+.$$

So, by the condition of the number$\xi_1 > 0$such that it is$\gamma(\xi_1) = 0$,set $A \cup B$is reduced to set C only by constraining component b vectors (a,b,c)which does not includevalues$\left[-\frac{(a-c)^2}{4}, 0\right]$.

Therefore, it is valid

$$C = \left\{(a,b,c) \in \left(\left[1,\infty) \times \left[-\frac{(a-c)^2}{4}, 0\right) \times (-\infty,-\pi)\right) \vee \right.\\ \left. (a,b,c) \in R\backslash\{0\} \times R^+ \times R^+\right\} \quad (4)$$

Next, let us examinethecondition $\xi_1 < \pi$.This condition is equivalent to therelationship

$$\sqrt{(a-c)^2 + 4b} > 2\pi + c - a$$

which is possible at $a < c + 2\pi$. Then it is

$$4b < 4\pi^2 + 4\pi(c-a)$$

or

$$b < \pi^2 + \pi(c - a)$$

therefore, these vectors $(a, b, c) \in C$whose components satisfy the relationship :

$$(a < c + 2\pi) \wedge b < \pi^2 + \pi(c - a)$$

subordinate the function γ for which it is valid $\xi_1 \in (0, \pi)$.Put

$$D = \{(a, b, c) \in C: (a < c + 2\pi) \wedge (b < \pi^2 + \pi(c - a))\} \quad (5)$$

We apply these considerations to solve the equation

$$-y''(x) + q(x)y(\gamma(x)) = \lambda y(x), \ \lambda = z^2 (6)$$

with the initial condition

$$y(\gamma(x)) = 0, \ x \in [0, \xi_1] \quad (7)$$

and the boundary condition at the left end of the segment $[0, \pi]$which is given with

$$y(0) - h(0) = 0, \ h \in R \quad (8)$$

The task (6-8) is equivalent to solving the integral Volter type equation

$$y(x, z) = cosxz + \frac{h}{z} sinxz + \frac{1}{z} \int_{\xi_1}^{x} q(t_1) sinz(x - t_1) y(\gamma(t_1), z) dt_1 \quad (9)$$

In order to solve the equation (9) by the method of variable steps, it is necessary to construct a strictly growing sequence $\xi_K, K > 1$, by recursive formula

$$\xi_K = \gamma(\xi_{K+1}), \qquad K = 1,2, \ldots.$$

or

$$\xi_{K+1} = \gamma^{-1}(\xi_K) (10)$$

It is necessary to select those vectors(a, b, c) from set D for which it is valid

$$\xi_K \in R, K = 2,3, \ldots [7]$$

For the sake of uniformity, we will put it

$$\xi_{-1} = -\frac{b}{c}, \ \xi_0 = 0, \text{so it is} \xi_1 = \gamma^{-1}(\xi_0).$$

Specifically, since γ is strictly growing on $[0, \pi]$, it is alsoγ^{-1} also rigorously growing in its domain. Forexample,settheconditionsforthevector(a, b, c) fromsetD,whichensuresthatthe number ξ_2 be in the distance (ξ_1, π).From$\xi_2 = \gamma^{-1}(\xi_1)$get

$$\xi_1 = \xi_2 - \frac{a\xi_2 + b}{\xi_2 + c}$$

or

$$(\xi_2)^2 + (c - a - \xi_1)\xi_2 - (b + c\xi_1) = 0$$

That is

$$(\xi_2)_{1,2} = \frac{1}{2}\left[a + \xi_1 - c \pm \sqrt{(a + \xi_1 - c)^2 + 4(b + c\xi_1)}\right]. \quad (11)$$

For

$$b + c\xi_1 \in \left[-\frac{(a + \xi_1 - c)^2}{4}, \ \infty\right], \text{worth it } \xi_2 \in R.$$

Thereby

$$\xi_2 = \frac{1}{2}\left[a + \xi_1 - c \pm \sqrt{(a + \xi_1 - c)^2 + 4(b + c\xi_1)}\right] > \xi_1.$$

Claim $\xi_2 < \pi$ leads to relationships

$$(a + \xi_1 - c < 2\pi) \wedge \ b < \pi^2 - \pi(a - c) - \xi_1(\pi + c).$$

From (10) followed by a similar relationship between two neighboring points ξ_K, ξ_{K+1}. She has a shape

$$\xi_{K+1} = \frac{1}{2}\left[(a - c + \xi_K) + \sqrt{(a - c + \xi_K)^2 + 4(b + c\,\xi_K)}\right], K \in N \qquad (12)$$

if it is $\xi_K \in R$ then it is $\xi_{K+1} \in R$ and in fact it is valid

$$b + c\,\xi_K \geq -\frac{(a - c + \xi_K)^2}{4} \qquad (13)$$

So, these vectors $(a, b, c) \in D$for which it is valid (13), $K = 2,3,$ ... generating a family of admissible functions of the type of bilineaarfunction.

Define numberK_0 as follows:

$$\xi_{K_0} < \pi \leq \xi_{K_0+1} \qquad (14)$$

Based on the numberK_0 family of permissible functions is divided into classes

$$\Gamma_{K_0}, \ K_0 = 1,2, ...$$

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