International Journal of Recent Engineering Research and Development (IJRERD)
ISSN: 2455-8761
www.ijrerd.com || Volume 06 – Issue 01 || January 2021 || PP. 08-14

Research on Numerical Simulation and LQR of Flexible Manipulator

JU Shijie[1], YAN Anzhi[1]
[1]*School of Civil Engineering, Henan Polytechnic University, Jiao'zuo 454000,China*

Abstract: At present, with the wide application of flexible manipulator, looking for advanced precise control method has become a prospective research direction. This paper takes the flexible manipulator as the research object. Firstly, according to the Euler Bernoulli beam model, the dynamic equation of the manipulator is derived. The first two modes and frequencies are obtained by ANSYS, which verifies the correctness of the theoretical calculation. Secondly, according to the modern control theory, the first two order state space of the lateral vibration of the piezoelectric flexible manipulator system is established, and the LQR control method is used to accurately control the manipulator. Finally, the influence of Q value and R value on the control effect of LQR is explored, and the control effect of traditional LQR and optimized LQR is compared.The simulation results show that the control accuracy of the optimized LQR control is 80% higher than that of the unoptimized LQR control, which indicates that the larger the Q is, the smaller the R is, the better the vibration suppression effect is, reflecting the scientific nature and necessity of LQR control research, and it has a broad application prospect in the field of structural vibration control.
Keywords: Flexible manipulator, ANSYS, state space, LQR control

1. Introduction

Flexible manipulators have the advantages of large working volume and low energy consumption, but they are prone to large deformation and low frequency vibration. With the development of modern control theory, how to control it accurately is a research hotspot [1],[2]. Piezoelectric materials (such as PZT, PVDF) attached to the surface of the flexible manipulator can use the piezoelectric effect to output voltage and exert force on the structure with the vibration of the structure, and the vibration control of the flexible manipulator can be realized through certain control strategies.

Zhong Wanxie [3] gave the analytical solution of Riccati differential equation based on the eigensolution of Hamiltonian matrix. For the Riccati differential equation controlled by LQR, he gave the formula based on Q value and R value, which provided the theoretical basis for LQR control of piezoelectric flexible manipulator. Wang Zongli [4] proposed a LQR control method based on state correlation, which adjusts the objective function and updates Q value according to the state at discrete time to control the piezoelectric flexible intelligent beam. Sun Jie [5] established a discrete form of rigid flexible coupling dynamic equation with MFC (piezoelectric fiber composite) piezoelectric drive by using Hamiltonian principle and load analogy method of piezoelectric drive, directly gave Q value and R value, and used LQR method for active control.

To sum up, when LQR control was used to suppress the vibration of flexible manipulator, the influence of Q value and R value on the control effect was not specifically studied. In this paper, according to the vibration characteristics of the piezoelectric manipulator, LQR control method is used to control the first two lateral vibration of the piezoelectric manipulator, and the influence of Q value and R value on the vibration suppression effect of LQR control is explored respectively. The results show that the larger the Q value is, the smaller the R value is, the better the vibration suppression effect is. The research results provide ideas for the optimal value of Q value and R value.

2. State space and numerical simulation of piezoelectric cantilever manipulator

2.1 State space of cantilever manipulator

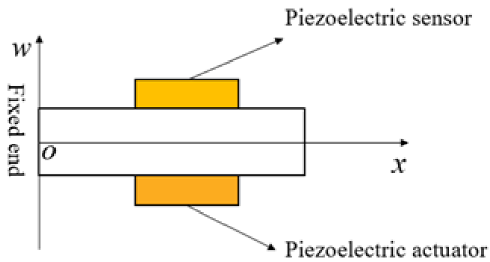


Figure 1: Structure of Piezoelectric Flexible Manipulator

As shown in Figure 1, the dynamic model of the piezoelectric flexible manipulator is established, and the differential equation of its lateral free vibration is as follows:

$$\rho A\frac{\partial^2 w}{\partial t^2}+EI\frac{\partial^4 w}{\partial x^4}=0 \quad (1)$$

Where, ρ, A, EI and w are the density, cross-sectional area, bending stiffness and deflection of the beam respectively.

According to the inverse piezoelectric effect of PZT, the moment generated by the voltage signal on the beam is as follows:

$$M(x,t)=K_\alpha u(t)[h(x-x_1)-h(\mathrm{x}-\mathrm{x}_2)] \quad (2)$$

Where, $h(x)$ is the step function $h(x-x_o)=\begin{cases}0 & x<x_o\\ 1 & x>x_o\end{cases}$ [6]; x_1 and x_2 are the left and right coordinates of the actuator; K_α [7] is the piezoelectric coupling coefficient, $K_\alpha=\frac{B_b d_{31}E_p(H_b+H_p)}{2}$ .where, B_b and H_b are the width and thickness of the manipulator, and d_{31}, E_p and H_p are the piezoelectric strain constant, elastic modulus and thickness of the piezoelectric sheet respectively.

The dynamic equation of the piezoelectric manipulator is as follows:

$$\rho A\frac{\partial^2 w}{\partial t^2}+EI\frac{\partial^4 w}{\partial x^4}=\frac{\partial^2}{\partial x^2}K_\alpha u(t)[h(x-x_1)-h(x-x_2)]=f_p(x,t) \quad (3)$$

At this time, the charge of the sensor is [7]:

$$Q(x,t)=\frac{1}{2}\int_{x_3}^{x_4}D\ (x,t)bdx=\frac{1}{2}\int_{x_3}^{x_4}d_{31}\sigma bdx \quad (4)$$

Where D is the potential shift and σ is the stress.

According to the PZT positive piezoelectric effect, the output voltage of the sensor is [6],[7]:

$$V(x,t)=\frac{Q(x,t)}{C_p} \quad (5)$$

Where C_p is the capacitance of the piezoelectric chip.

The deflection is expressed in modal coordinates as follows:

$$w(x,t)=\sum_{i=1}^{n}\varphi_i(x)q_i(t)=\Phi^T q(t) \quad (6)$$

Where, $\varphi_i(x)$ and $q_i(t)$ represent the second mode shape and coordinate vector of the system respectively;

The expression of the i th mode $\varphi_i(x)$ of the piezoelectric manipulator is as follows:

$$\varphi_i(x) = [\cos\beta_i x - \cosh\beta_i x + \gamma_i(\sin\beta_i x - \sinh\beta_i x)] \quad (7)$$

Where, $\beta_1 = \frac{1.875}{L_b}$, $\beta_2 = \frac{4.694}{L_b}$, $\gamma_i = -\frac{\cos\beta_i L_b + \cosh\beta_i L_b}{\sin\beta_i L_b + \sinh\beta_i L_b}$ (8)

According to equation (6), the vibration differential equation of the i th mode of the piezoelectric manipulator is obtained:

$$\ddot{q}_i(t) + 2\xi_i\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = \int_0^{l_b} \varphi_i(x) f_p(x,t)dx \quad (9)$$

Where, ξ_i and ω_i represent the damping ratio and natural frequency of the i th mode of the manipulator, respectively, and ω_i is:

$$\omega_i = (\beta_i L_b)^2 \sqrt{\frac{E_b I}{\rho_b S_b L_b^4}} \quad (10)$$

Where, S_b and I represent the cross-sectional area and the moment of inertia of the manipulator respectively.

The state vector is introduced:

$x(t) = [q \quad \dot{q}]^T = [q_1(t) \quad q_2(t) \quad \cdots \quad q_{n(t)} \quad \dot{q}_1(t) \quad \dot{q}_2(t) \quad \cdots \quad \dot{q}_n(t)]^T$,so state space of piezoelectric manipulator can be expressed as:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (11)$$

Where, A represents the state matrix; B represents the control matrix; C represents the output matrix; $y(t)$ represents the output voltage; $u(t)$ represents the input voltage. The specific expression is as follows:

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -\Omega^2 & -2\Lambda \end{bmatrix},$$

$$\Omega = diag[\omega_1 \quad \omega_2 \quad \cdots \quad \omega_n],$$

$$\Lambda = diag[\xi_1\omega_1 \quad \xi_2\omega_2 \quad \cdots \quad \xi_n\omega_n],$$

$$B = [0_{1\times n} \quad \tilde{B}_1 \quad \tilde{B}_2 \quad \cdots \quad \tilde{B}_n]^T,$$

$$C = [\tilde{C}_1 \quad \tilde{C}_2 \quad \cdots \quad \tilde{C}_n \quad 0_{1\times n}],$$

$$\tilde{C}_i = K_\beta[\varphi_i'(x_4) - \varphi_i'(x_3)].$$

Where, $K_\beta = \frac{B_b H_b d_{31} E_p}{2C_p}$.

2.2 Modal analysis of manipulator

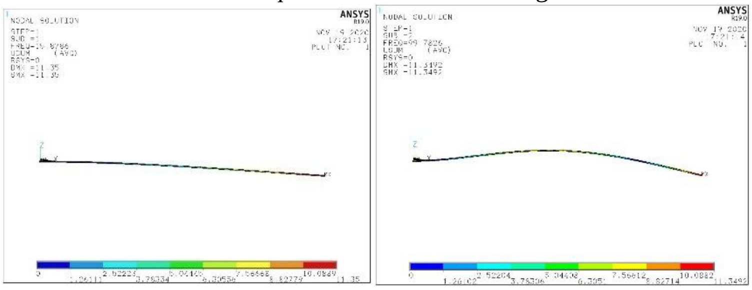
ANSYS finite element software is used to simulate the vibration mode of the manipulator, and the parameters are shown in Table 1:

Table 1: Parameters of mechanical arm and piezoelectric plate

physical property	Mechanical arm	physical property	PZT piezoelectric sheet
Young's modulus E	7.3×10^{10} pa	Young's modulus E_p	7.65×10^{10}pa
density ρ	2700 kg/m^3	density ρ_p	7450 kg/m^3
Poisson's ratio μ_b	0.3	Poisson's ratio μ_p	0.3

length L_b	0.23 m	length L_p	0.03 m
width B_b	0.05 m	width B_p	0.002 m
thickness H_b	0.001 m	thickness H_p	0.002 m
First / second order damping ratio	0.007	Piezoelectric constant d_{31}	-1.85×10^{-10}m/V
		capacitance C_p	79nF
		First / second order damping ratio	0.007

The obtained vibration modes and frequencies are shown in Figure 2 and table 2:



（a） First order （b） Second order

Figure 2: First two modes

Table 2: First and second order frequency values

Order	ANSYS simulation	numerical calculation
1	15.879 Hz	15.88 Hz
2	99.783 Hz	99.55 Hz

It can be concluded from table 2 that the difference between the simulated frequency and the calculated frequency is not more than 1%, which proves the accuracy of the theoretical calculation.

By substituting the data into equation (11), the state space model of the piezoelectric manipulator can be obtained as follows:

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9951.1 & 0 & -1.3966 & 0 \\ 0 & -390876 & 0 & -8.7528 \end{bmatrix} B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.039 & -0.0016 \\ -0.0083 & 0.074 \end{bmatrix}$$

$$C=\begin{bmatrix} -16256 & -77146 & 0 & 0 \\ -6906.5 & 75284 & 0 & 0 \end{bmatrix} \quad (12)$$

3. LQR control algorithm and simulation analysis

3.1 LQR control algorithm

The objective function of LQR control is [4]:

$$J=\int_0^{\infty}(x^TQx+Ru^2)dt \quad (13)$$

Where: Q and R are positive semi definite state matrix and positive definite control weight matrix respectively.

The control input which minimizes the objective function J can be expressed as:

$$u=-Gx \quad (14)$$

Where, $G = R^{-1}B^{T}K$ is gain matrix, K is the solution of Riccati equation:

$$KA + A^{T}K + Q - KBR^{-1}B^{T}K = 0 \quad (15)$$

Note that the choice of Q and R is not fixed.

3.2 Research on LQR control simulation

3.2.1 Simulation of traditional LQR control

Give a unit step load to the manipulator, and choose $Q = \begin{bmatrix} 1e9 & 0 & 0 & 0 \\ 0 & 1e9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $R = [100]$,according to the experience. The simulation results are shown in Figure 3

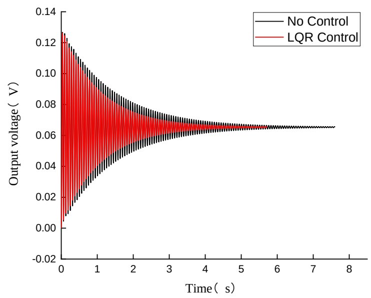


Figure 3: LQR output voltage

It can be seen from Figure 3 that the existing values of Q and R make the LQR control effect not very good, and the control accuracy is only improved by about 5%. Further optimization is needed to explore the influence of the values of Q and R on the LQR control effect, so as to greatly improve the control accuracy.

3.2.2 Explore the influence of 1 and 2 values respectively

$Q_1 = \begin{bmatrix} 1e9 & 0 & 0 & 0 \\ 0 & 1e9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $Q_2 = \begin{bmatrix} 1e10 & 0 & 0 & 0 \\ 0 & 1e10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $Q_3 = \begin{bmatrix} 1e11 & 0 & 0 & 0 \\ 0 & 1e11 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for Q, and $[100]$ for R. The simulation results are shown in Figure 4.

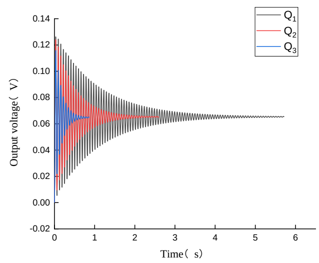


Figure4: Influence of Q value on LQR control effect

It can be seen from Figure 4 that the LQR control effect increases with the increase of Q matrix when R remains unchanged. For each order of magnitude increase of Q, the control effect increases linearly by about 50%, so Q should be large enough.

In the same way. Let R take $\begin{bmatrix} 1e9 & 0 & 0 & 0 \\ 0 & 1e9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, R take $R_1 = [100]$, $R_2 = [10]$ and $R_3 = [1]$ respectively. The simulation results are shown in Figure 5.

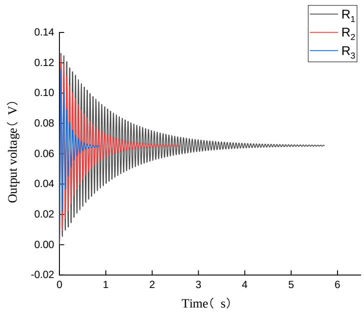


Figure5: Influence of R value on LQR control effect

It can be seen from Figure 5 that the LQR control effect increases with the decrease of R matrix when Q remains unchanged. Every time R decreases by an order of magnitude, the control effect increases linearly by about 50%. Therefore, R should be small enough. Here, we only discuss the influence of R and R values on LQR control effect, without considering the energy factor.

In conclusion, the larger the Q is $\begin{bmatrix} 1e11 & 0 & 0 & 0 \\ 0 & 1e11 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, the smaller the R is $[1]$, the better the vibration suppression effect is. Therefore, the control effect should be optimal when Q is chosen and R is chosen. The simulation results are shown in Figure 6.

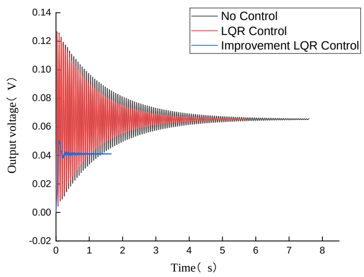


Figure6: optimizing Q 、 R output voltage

As can be seen from Figure 6, the optimized LQR control effect not only greatly shortens the control time, reduces 80% and 60% respectively compared with the uncontrolled and unoptimized LQR control, but also greatly improves the control accuracy, making the voltage amplitude and stability amplitude reduce 60% and 33% respectively compared with the unoptimized LQR control, which further verifies the effectiveness of LQR control and the correctness of the conclusion.

4. Conclusions

(1). According to Euler Bernoulli beam model and modern control theory, the first two state space models of flexible manipulator are established.

(2). The modal analysis of flexible manipulator is carried out by ANSYS software, and the accuracy of theoretical calculation is verified.

(3). LQR control is used to control the vibration of the flexible manipulator, and the conclusion that the larger the Q is, the smaller the R is, the better the vibration suppression effect is obtained. The results show that it is scientific and necessary, and provide a reference for finding the optimal value of Q and R.

(4). The relationship between the change of Q and R values and the energy consumption and control accuracy will be further studied in the future.

References

[1] Shi Chao. Multi time scale sliding mode control of flexible manipulators [J]. Control engineering, 2020,27 (09): 1560-1565.

[2] Jia Zhen, Lou Junqiang, Yang Yiling, et al. Experimental model identification and servo speed vibration suppression control of rotating flexible manipulator system [J]. Vibration and impact, 2020,39 (24): 76-82 + 133.

[3] Zhong Wanxie. Analytical solutions of matrix Riccati differential equations [J]. Mechanics quarterly, 2000 (01): 1-7.

[4] Wang Zongli, Lin Qirong, Liu Zhengxing. State dependent LQR vibration control of piezoelectric smart beams [J]. Journal of Shanghai Jiaotong University, 2001 (04): 503-508.

[5] Sun Jie, Huang Tingxuan, Zhu Dongfang, et al. Dynamic modeling and vibration suppression of Spacecraft Based on piezoelectric fiber composites [J]. Flight control and detection, 2019,2 (03): 70-76.

[6] GOH C J, CAUGHEY T K. On the stability problem caused by finite actuator dynamics in the control of large space structures [J]. Int J Control, 1985, 41:787-802.

[7] UMAPHHY M., BANDYOPADHYAY B. Vibration control of flexible beam through smart structure concept using periodic output feedback[J]. Proceedings of System Science, 2003: 23-46.