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Energy-Momentum Relation and Eigen Equations In a Curved Space Time

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Abstract: Using the expression of time and distance in a curved space time a useful expression of energy and momentum Eigen equation similar to that is a curved space is found. These relations can be used to derive the corresponding relations in the Euclidean space. The corresponding on of Energy-Momentum relations for both curved and Euclidian space gives a relation between energy and momentum typical to that obtained from the energy and momentum Eigen equations. The expression of mass in a curved space is similar to that of the generalized relativity.

Keywords: Curved space- time, Energy Eigen equation, Momentum Eigen equation, Energy-Momentum relation.

Introduction

Atoms are building block of matter. Atoms themselves consist of elementary particles like electrons, protons and neutrons [1]. The behavior of atoms described by quantum lows. The one which is biased the classical Newtonian energy-momentum relationship is known as Schrodinger equation [2]. That who riles on relativistic energy-momentum relation is known as Klien-Gordan and Dirac relativistic quantum equations [3].Fortunately a quantum law successfully describes a wide variety of phenomena, like atomic spectra of isolated atoms beside the spectra of some solids like semi-conductor (SC). However quantum mechanics (qm) suffer from noticeable setbacks [4]. For instant there is no quantum gravity low that moreover the unification of force under the umbrella of quantum low is so difficult within the framework of conventional quantum lows [6].Also the superconductivity behavior for high temperature superconductors (HTSC) can hot be described easily and fully by the existing models [7]. This forces many researchers to construct new models that modify quantum lows to cure some of these defects [8, 9].These attempts encourages to purpose quantum model that can help in finding quantum gravity equation. This model was an expression of the wave function in a curved space to find energy and momentum Eigen equations in a curved space.

The energy within the framework of the GSR and SR are given by:

$$g_{00}E^2 = g_{xx}P^2c^2 + g_{00}^2m_0^2c^4 \ , \ E_0^2 = P_0^2c_2 + m_{02}c^4 \quad (1)$$

Where E_0 is the ordinary SR energy.
Thus the GSR energy E is given by

$$E = g_{00}^{-\frac{1}{2}}E_0 \quad (2)$$

The wave function in the curved space is thus

$$\psi = Ae^{\left(\frac{i}{\hbar}\right)(Px_c - Et_c)} \quad (3)$$

Energy Eigen equation and time independent Schrodinger equation in the Euclidean space takes the form

$$i\,\hbar\frac{\partial\psi}{\partial t} = E_0\psi \quad (4)$$

Also the momentum Eigen equation in the Euclidian space is given by

$$\frac{\hbar}{i}\nabla\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial x} = P_0\psi \quad (5)$$

In a curved space GSR wave function for free particle is given by

$$\psi = Ae^{(\frac{i}{\hbar}(\sqrt{g_{xx}}P_x - \sqrt{g_{00}}\,Et)}$$

Where $dt_c = \sqrt{g_{00}}dt \qquad dx_c = \sqrt{g_{xx}}dx$ (6)

Schrodinger equation in the curved space, where the time is denoted byt_c, can read

$$i\hbar\frac{\partial\psi}{\partial t_c} = i\hbar\frac{\partial\psi}{\partial t_c} = i\hbar\frac{\partial\psi}{\partial\sqrt{g_{00}}dt} = \frac{i\hbar}{\sqrt{g_{00}}}\left[-\frac{i}{\hbar}\sqrt{g_{00}}\,E\psi = \frac{i\hbar}{\sqrt{g_{00}}}\frac{\partial\psi}{\partial t}\right] = \frac{1}{\sqrt{g_{00}}}E\psi = \frac{\sqrt{g_{00}}}{\sqrt{g_{00}}}E\psi = \frac{\sqrt{g_{00}}}{\sqrt{g_{00}}}E\psi = E\psi \quad (7)$$

Thus

$$i\,\hbar\frac{\partial\psi}{\partial t_c} = E\psi \quad (8)$$

But from (3)

$$i\,\hbar\frac{\partial\psi}{\partial t_c} = E\psi \quad (9)$$

This is completely consistent with equation (8). Conversely from (6), (9) and (2)

$$i\,\hbar\frac{\partial\psi}{\partial t_c} = i\,\hbar\frac{\partial\psi}{\partial\sqrt{g_{00}}dt} = E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = \sqrt{g_{00}}E\psi = E_0\psi$$

$$E_0 = \sqrt{g_{00}}E \quad (10)$$

The momentum Eigen equation for the momentum in Euclidean space takes the form

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\psi = P_0\psi \quad (11)$$

In a curved space, the momentum Eigen equation becomes

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial x_c} = P\psi$$

With

$$dx_c = \sqrt{g_{xx}}dx \quad (12)$$

Thus

$$\frac{\hbar}{i}\frac{\partial\psi}{\sqrt{g_{xx}}dx} = P\psi \quad (13)$$

Thus, one can write

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\psi = \sqrt{g_{xx}}\,P\psi \quad (14)$$

Comparing this relation with (11) yields

$$P_0 = \sqrt{g_{xx}}\,P \quad (15.a)$$

Where

$$P_0 = \sqrt{g_{xx}}P \quad (15.b)$$

Thus equations (14) and (15.a) gives

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\psi = P_0\psi \quad (15.c)$$

This is the ordinary momentum Eigen equation in the Euclidian space.

The velocity in a curved space is define to be

$$v = \frac{\sqrt{g_{xx}}}{\sqrt{g_{00}}}\frac{dx}{dt} = \frac{\sqrt{g_{xx}}}{\sqrt{g_{00}}}v_0 \quad (16)$$

But the momentum in a curved space and Euclidean

$$P = mv$$

$$P_0 = m_0 v_0 \quad (17)$$

Using equation (15)

$$P = (g_{xx})^{\frac{1}{2}}P_0$$

Thus

$$mv = (g_{xx})^{-1}m_0 v_0$$

$$m = \frac{\sqrt{g_{00}}}{(\sqrt{g_{xx}})^2}m_0 = \frac{\sqrt{g_{00}}}{g_{xx}}m_0 \quad (18)$$

Since in driving GSR, one assumes that

$$g_{xx} = 1 \quad (19)$$

It follows that

M=$g_{00}m_0$ (20)

But the mass in GSR is given by

$$m = \frac{g_{00} m_c}{\sqrt{g_{00} - v^2/c^2}} \tag{21}$$

For the mass at rest

$v = 0$

$$m = \frac{g_{00} m_c}{\sqrt{g_{00}}} = \sqrt{g_{00}} m_0 \tag{22}$$

This relation is consistent with equation.

To find the expression, which relates E to P in a curved space-time one uses the relation

$$c^2 dt^2 = c^2 g_{00} dt_0^2 - g_{xx} dx^2$$

$$\gamma^{-1} = \left(\frac{d\tau}{dt}\right) = \left[g_{00} - g_{xx} \frac{v_0^2}{c^2}\right]^{\frac{1}{2}} \tag{23}$$

Thus

$$E = mc^2 = g_{00} \gamma m_0 = \frac{g_{00} m_0 c^2}{\sqrt{g_{00} - g_{xx} \frac{v_0^2}{c^2}}} \tag{24}$$

But from (16)

$$g_{xx} v_0^2 = g_{00}^2$$

$$E = \frac{g_{00} m_0 c^2 E}{\sqrt{g_{00} E^2 - g_{xx} P^2 c^2}}$$

$$g_{00} E^2 - g_{00} P^2 c^2 = g_{00}^2 m_0^2 c^4$$

$$g_{00} E^2 = g_{00} P^2 c^2 + g_{00}^2 m_0^2 c^4 \tag{25}$$

Setting

$$E_0^2 = g_{00} E^2 P_0^2 = g_{00} P^2 \tag{26}$$

One gets

$$E_0^2 P_0^2 c^2 + m_0^2 c^4 \tag{27}$$

However, when one replaces v_0 by v in equation (23), one gets

$$\gamma^{-1} = \left[g_{00} - g_{xx} \frac{v_0^2}{c^2}\right]^{\frac{1}{2}} \tag{28}$$

As a result, energy becomes

$$E = \frac{g_{00} m_0 c^2}{\sqrt{g_{00} - g_{xx} v^2/c^2}} = \frac{g_{00} m_0 c^2}{\sqrt{\frac{g_{00} m^2 c^4 - g_{xx} m^2 v^2 c^2}{m^2 c^4}}}$$

$$= \frac{g_{00} m_0 c^2}{\sqrt{\frac{g_{00} E^2 - g_{xx} P^2 c^2}{E^2}}}$$

$$E = \frac{g_{00} m_0 c^2 E}{\sqrt{g_{00} E^2 - g_{xx} P^2 c^2}}$$

$$g_{00} E^2 - g_{xx} P^2 c^2 = g_{00}^2 m_0^2 c^4$$

$$g_{00} E^2 = g_{xx} P^2 c^2 + g_{00}^2 m_0^2 c^4 \tag{29}$$

By setting

$$E_0^2 = g_{00} E^2 P_0^2 = g_{xx} P^2$$

$$E_0 = g_{00}^{\frac{1}{2}} E \qquad P_0 = \sqrt{g_{xx}} P \tag{30}$$

One gets

$$E_0^2 = P_0^2 c^2 + \widetilde{m_0}^2 c^4 \tag{31}$$

Where

$$\widetilde{m_0} = g_{00} m_0 \tag{32}$$

Harmonic Oscillator in a Curved Space:

The Schrodinger equation in a curved space is given

$$i\hbar \frac{\partial \psi}{\partial t_c} = E\psi \qquad i\hbar \frac{\partial \psi}{\partial \sqrt{g_{00}}} = E\psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = \sqrt{g_{00}}\, E\psi = E_0 \psi \tag{33}$$

For harmonic oscillator in Euclidean space

$$E_0 = \left(n + \frac{1}{2}\right)\hbar\omega \tag{34}$$

$$(1+x)^n \approx 1 + nx$$

For $x<1$

$$\frac{2\emptyset}{c^2} = x \tag{35}$$

$$(g_{00})^{-\frac{1}{2}} = \left(1 + \left(-\frac{1}{2}\right)\frac{2\emptyset}{c^2}\right)$$

$$(g_{00})^{-\frac{1}{2}} = \left(1 - \frac{\emptyset}{c^2}\right) \tag{36}$$

This approximation is justifiable since $\frac{2m\emptyset}{mc^2} < 1 \qquad 2V < mc^2$

Which means that the total energy is the greater than potential energy. Thus equation (30) gives

$$E = E_0 (g_{00})^{-\frac{1}{2}} = E_0\left(1 - \frac{\emptyset}{c^2}\right) \tag{37}$$

$$E = \left(n + \frac{1}{2}\right)\left(1 - \frac{\emptyset}{c^2}\right)\hbar\omega$$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{\emptyset}{c^2}\left(n + \frac{1}{2}\right)\hbar\omega \tag{38}$$

$$E = E_0\left(1 - \frac{\emptyset}{c^2}\right) = E_0\left(1 - \frac{m_0\emptyset}{m_0 c^2}\right) = E_0\left(1 + \frac{V_0}{E_0}\right) = E_0 + V_0 \tag{39}$$

Discussion

The wave function in a curved time-space is given by equations (1) and (6). Using this expression together with definition of time in a curved space-time in equation (10), the curved space- Eigen equation take a form typical to Euclidian space, this is very apparent when comparing equation (5) and (7). It is very striking to note that the energy Eigen equation in a curved space-time [equation (9)] can be used to derive the energy Eigen equation in the Euclidean space. The same hold for momentum Eigen equation in a curved space in equation (12) can be used to derive that Euclidian space in equation (15.c). Fortunately, the relation between the momentum in a curved space and Euclidean space is typical to that in equation (1).

Using the expression for the velocity and momentum and in free space and a curved space-time as shown in equations (16) and (17). A useful expression for a mass in a curved space-time in terms of rest mass is found in equation (22). It is very striking note that the expression is similar to that of GSR. The energy-momentum relation in a curved space-time found in equation (29) gives curved space-time, Euclidian analogy between energy and momentum in equation (30) is typical to that obtained for Eigen equations in equations (10) and (15.a).

Applying energy Eigen equation for harmonic oscillator in a curved space-time shows that the energy in the curved space-time is equivalent to the existence of additional potential term typical to that Newton. This means that a particle in a curved space behavior is typical to the behavior of a particle moving in a potential.

Conclusion

The energy and momentum Eigen equation in a curved space can be used to derive that the Euclidean space using the energy-momentum relation analogy in Euclidian and curved space. It the expression of mass in a curved space similar to that of GSR.

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