International Journal of Recent Engineering Research and Development (IJRERD)
ISSN: 2455-8761
www.ijrerd.com || Volume 04 – Issue 10 || October 2019 || PP. 01-12

Empirical Comparison of Relative Precision of Geometric Measure of Variation about the Mean and Standard Deviation

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Abstract: Measures of dispersion are important statistical tool used to illustrate the distribution of datasets. These measures have allowed researchers to define the distribution of various datasets especially the measures of dispersion from the mean. Researchers have been able to develop measures of dispersion from the mean such as mean deviation, mean absolute deviation, variance and standard deviation. Studies have shown that standard deviation is currently the most efficient measure of variation about the mean and the most popularly used measure of variation about the mean around the world because of its fewer shortcomings. However, studies have also established that standard deviation is not 100% efficient because the measure is affected by outlier in the datasets and it also assumes symmetry of datasets when estimating the average deviation about the mean a factor that makes it to be responsive to skewed datasets hence giving results which are biased for such datasets. The aim of this study is to make a comparative analysis of the precision of the geometric measure of variation and standard deviation in estimating the average variation about the mean for various datasets. The study used paired t-test to test the difference in estimates given by the two measures and four measures of efficiency (coefficient of variation, relative efficiency, mean squared error and bias) to assess the efficiency of the measure. The results determined that the estimates of geometric measure were significantly smaller than those of standard deviation and that the geometric measure was more efficient in estimating the average deviation for geometric, skewed and peaked datasets. In conclusion, the geometric measure was not affected by outliers and skewed datasets, hence it was more precise than standard deviation.

Keywords: Standard Deviation, Geometric Measure of variation, deviation about the mean, average, mean, measure of efficiency.

Introduction

Geometric measure of variation about the mean is a new measure of variation about the mean which is created on the basis of solving the weaknesses of current measure of variation about the mean such as violation of the algebraic laws governing absolute numbers for the case of mean deviation, a factor which has made the measure not to be precise in terms of estimation and also prevent any further algebraic manipulation from being carried out on the measure[10],[2]. Variance has been determined to give results on average squared deviation from the mean which are of different units as the original datasets (not squared) a factor which makes the results inappropriate [8]. Standard deviation hasalso been determined to be affected by outliers and skewed datasets [3], [8].

Amongcurrent existing measures of variation about the mean, standard deviation has popularly been used by many researchers to the extent that it has been widely accepted as the ultimate measure of dispersion from the mean. This is because of its superiority in terms of fewer number of shortcomings compared to the other measures of dispersion from the mean. The improvements on the other measures of variation about the mean brought about standard deviation, a factor which has made the measure to be considered as the ultimate measure of variation about the mean[1], [11], [8-9].

Standard deviation has growingly been improved on from its use on un-weighted datasets to its use in weighted datasets. From its use on population datasets to its use on the sub-sample of the populations. From its use in only on quantitative datasets to its use on proportions in qualitative data and also its use on distribution functions through moment generating functions. All these factors and improvements has enabled the function to be recognized as the internationally known measure of dispersion from the mean regardless of its limitations such as response to outliers and skewed datasets. As a result, any new measure of dispersion from the mean that can be invented must be tested against standard deviation because it is the ultimate measure of dispersion from the mean and the internationally recognized measure of average deviation from the mean. According to trend on invention of new measures of dispersion, the new measures are developed by improving on the weaknesses of the existing measures [9], [1], [11].

The aim of this study is to compare the relative precision of the geometric measure of variation about the mean and standard deviation in estimating the average deviation about the mean for various datasets especially datasets with outliers and skewed datasets, so as to check if the geometric measure of variation is capable of mending the problems of standard deviation which is response to outliers and skewed datasets.

Methods

Comparison of Deviation Results

The study compared the results of average deviation from the mean given by the geometric measure of deviation from the mean and standard deviation for populations of size 200 and 4000 un-weighted observation. The study simulated the observations using MINITAB version 17 software, after which the dataset was divided into 20 random samples each of size 10 and 200 respectively, to represent small and large samples. The results of each sampleyield one measure of geometric deviation from the mean and one standard deviation, hence the 20 samples yield a total of 20 geometric deviations from the mean and 20 standard deviation values for small and large samples each.

The 20 geometric deviation values constitute the first sample group and the 20 standard deviation values constitute the second sample values, for both small samples of size 10 and large samples of size 200. These are average deviation from the means which constitute the two samples which were considered as usual average values and a paired sample t-test was used to determine if there is a significant difference on the two average set of values and also if the results for the geometric mean are significantly less than those of standard deviation.

Let$S = [s_1, s_2, s_3, \ldots, s_{20}]$be the set of standard deviation for the ten samples and$G = [g_1, g_2, g_3, \ldots, g_{20}]$ be the set of geometric deviation for the ten samples. Let$\nabla_i = g_i - s_i$which is the difference between the geometric deviation and standard deviation of sample i. Letσ_∇ to be the standard deviation of the difference between the geometric deviation and standard deviation of all the 20 samples. The test statistics for the paired sample t-test was given by the formula;

$$t = \frac{\overline{\nabla}}{\sqrt{\frac{\sigma_\nabla^2}{20}}} \quad (1)$$

Where;

$$\overline{\nabla} = \frac{\sum_{i=1}^{20} \nabla_i}{20} \quad (2)$$

And,

$$\sigma_\nabla^2 = \frac{\sum_{i=1}^{20} \left(\nabla_i - \overline{\nabla}\right)^2}{20} \quad (3)$$

Two-sided t-test was carried out at 95% level of confidence based on the following null and alternative hypothesis;

$$H_0: Thetwomethodsgivessimilarresults$$

Against

$$H_1: Thetwomethodsdonotgivesimilarresults$$

If the above test results to the rejection of the null hypothesis, then another one-sided test was carried out at 95% level of confidence based on the following null and alternative hypothesis;

$$H_0: \bar{G} = \bar{S}$$

Against

$$H_1: \bar{G} < \bar{S}$$

If the results yield to the rejection of the null hypothesis, then the geometric deviation was deemed to yield smaller estimates of deviation from the mean as compared to standard deviation.

Testing for Efficiency of the Method

The efficiency of the geometric measure of variation from the mean was tested against standard deviation technique to determine the most efficient estimation technique of the average deviation from the mean. The study checked at the most efficientmeasure based on then Mean Squared Error (MSE), Bias, Relative efficiency and Coefficient of Variation of the geometric measure of variation about the mean in comparison to standard deviation;

i. Mean Squared Error

Consider a population of set of data points V such that $V = v_1, v_2, \ldots, v_N$, and let d be a set of deviations from the mean such that $d = d_1, d_2, \ldots, d_N$ where $d_i = v_i - \bar{V}$, where $\bar{V}$ is the population mean. Let G be the geometric measure of variation from the mean for the population and σ be the population standard deviation.

A total of k samples were selected from the population of size N with geometric measure of variation from the mean G and Standard deviation σ. For each sample, an estimate of average deviation about the meanwas calculate using the geometric measure of variation from the mean and also using standard deviation, this yield a total of k estimates for the geometric measure and similar k estimates for the standard deviation. The average of the k geometric estimates of the average deviation from the mean was given by;

$$\bar{g} = \frac{\sum_{i=1}^{k} g_i}{k} \quad (4)$$

Where, g_i is the geometric measure of variation from the mean for the i[th] sample which is given by the formula;

$$g_i = \begin{cases} \exp\left(\frac{1}{n-1}\sum_{i=1}^{P}\ln(|d_i|)\right) & \forall d_i \neq 0 \\ 0 & \forall d_i = 0 \end{cases} \quad (5)$$

Similarly, the mean of the standard deviation estimators was given by;

$$\bar{S} = \frac{\sum_{i=1}^{k} S_i}{k} \quad (6)$$

Where,

$$S_i = \frac{\sum_{i=1}^{n}(v_i - \bar{v})^2}{n-1} \quad (7)$$

The mean squared error for geometric measure of variation from the mean was given by;

$$MSE = \frac{\sum_{i=1}^{k}(g_i - G)^2}{k} \quad (8)$$

Where,

$$G = \begin{cases} \exp\left(\frac{1}{n}\sum_{i=1}^{P}\ln(|d_i|)\right) & \forall d_i \neq 0 \\ 0 & \forall d_i = 0 \end{cases} \quad (9)$$

Similarly, for standard deviation estimates the mean squared error was given by;

$$MSE = \frac{\sum_{i=1}^{k}(S_i - \sigma)^2}{k} \quad (10)$$

Where,

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(v_i - \bar{v})^2}{n}} \quad (11)$$

The MSE results for equation 8was compared to that of 10, the estimation technique that gave result into a smaller MSE was said to be more efficient estimator than the other.

ii. Bias

Bias refers to the difference between the estimated value and the parameter value in point estimation. During the estimation of the average from the mean using either the geometric measure of deviation from the mean or standard deviation, the bias was given by the difference between the population parameters G and σ, against the average of the sample estimates $\bar{g}$ and $\bar{S}$, respectively. For the geometric measure of deviation from the mean, the bias was given by;

$$BIAS = \bar{g} - G \quad (12)$$

Similarly, for standard deviation the bias was given by;

$$BIAS = \bar{S} - \sigma \quad (13)$$

The estimation method which had higher bias based on equation 12or13was deemed less efficient than the other.

iii. Relative Efficiency

The efficiency of the geometric measure of variation was tested against the standard deviation using the relative efficiency. Relative efficiency gives the ratio of the variance of the two estimation techniques of the average deviation from the mean. Based on the p samples, the variance of the geometric measure of deviation for the samples was calculated by;

$$Variance(g) = \frac{\sum_{i=1}^{k} (g_i - \bar{g})^2}{k} \quad (14)$$

Similarly, the variance of the estimates given by standard deviation was given by;

$$Variance(S) = \frac{\sum_{i=1}^{k} (S_i - \bar{S})}{k} \quad (15)$$

based on the results calculated in 14 and 15 the relative efficiency of geometric measure of variation in relation to standard deviation was calculated by;

$$\text{Relative Efficiency} = \frac{Variance(S)}{Variance(g)} \quad (16)$$

If the results of 16 is determined to be less than 1 then the standard deviationwas deemed more efficient than geometric measure. If the result is determined to be more than 1 then standard deviation was considered less efficient than the geometric measure of variation. Otherwise, if the result is 1 then the two measure was deemed to be equally efficient in estimating the average variation from the mean.

iv. Coefficient of Variation

Coefficient of variation was used to check at the average spread of each estimate given by each of the estimation technique from the mean. This was help in illustrating the efficiency of each of the individual estimation technique in giving estimated which are closer to the true estimation value. An estimation technique which was results into a lower coefficient of variation compared to the other was considered as less efficient. For the geometric measure of variation from the mean, the coefficient of variation was given by;

$$CV(g) = \frac{\sqrt{\mathrm{var}(g)}}{\bar{g}} \quad (17)$$

Similarly, the coefficient of variation of standard deviation was calculated by;

$$CV(S) = \frac{\sqrt{\mathrm{var}(S)}}{\bar{S}} \quad (18)$$

The results for equation 17was compared to that of 18 to determine the most efficient estimator in term of giving estimates which are closer to the mean value. The equation that resulted into a smaller coefficient of variation was considered more efficient than the other.

Results

Paired sample t-test was used to test for significant difference in the sample estimates obtained by geometric deviation in comparison to the estimates obtained by standard deviation. The test was also used to check if the geometric estimates were significantly smaller than those of standard deviation. In order to assess the efficiency of the geometric measure about the mean, four measures of efficiency were used to assess the efficiency of the function (Mean Squared Error, Coefficient of variation, Relative Efficiency and Bias). The study used both small samples of size 10 each and large samples of size 200 each to assess the efficiency. The results were as illustrated below.

i. Small Geometric Discrete Sample

Twenty Geometric distributed samples each of size 20 with 0.5 probability of success were simulated, the estimates for geometric measure of variation about the mean and standard deviation for each of the samples were as illustrated in table 1;

Sample Number	Standard Deviation	Geometric Measure of Variation
1	0.632	0.363
2	2.003	1.147
3	1.729	1.189
4	3.651	1.661
5	0.876	0.494
6	1.506	0.819
7	2.573	1.075
8	1.080	0.776
9	0.823	0.614
10	1.287	0.873
11	1.449	0.777
12	0.527	0.500
13	1.776	1.293
14	0.699	0.555
15	1.075	0.720
`16	1.229	0.839
17	1.491	1.196
18	1.197	0.339
19	0.823	0.614
20	1.054	1.000

Table 1

The test for significant difference between the standard deviation measures and the geometric measure of variation showed that there was a significant difference between the geometric measures estimates and the standard deviation measure ($t=4.9274$, $df=19$, P-value<0.001). the test showed that the average estimates for the geometric measure of variation from the mean were significantly smaller than those of standard deviation (P-value<0.001).

The test results for efficiency of the geometric measure in comparison to standard deviation, based on the four measures of efficiency were as illustrated in table 2;

Measure of Efficiency	Standard Deviation	Geometric Measure of Variation
Coefficient of Variation	0.5348	0.4062

Relative Efficiency	4.615	
Bias	0.186	0.087
Mean Squared Error	0.548	0.119

Table 2

The results in the above table illustrate that Geometric Measure of variation from the mean was more efficient than standard deviation. Geometric Measure of variation had a smaller coefficient of variation meaning that the estimates were close to each other (smaller standard deviation), the variance of Geometric Measure of variation are smaller than those of standard deviation, this makes Geometric Measure of variation to have a relative efficiency smaller than that of standard deviation. In terms of bias, Geometric Measure of variation had a smaller bias compared to the standard deviation. Lastly, the mean squared errors for Geometric Measure of variation was also smaller than that of standard deviation. Therefore, for small geometric samples, Geometric Measure of variation is a more efficient measure of variation from the mean than standard deviation.

Large Geometric Discrete Sample

Twenty Geometric distributed samples each of size 200 with 0.5 probability of success were simulated, the estimates for geometric measure of variation about the mean and standard deviation for each of the samples were as illustrated in table 3;

Sample Number	**Standard Deviation**	**Geometric Measure of Variation**
1	1.293	0.694
2	1.522	0.631
3	1.268	0.462
4	1.252	0.415
5	1.497	0.479
6	1.433	0.558
7	1.422	0.594
8	1.326	0.489
9	1.447	0.465
10	1.405	0.256
11	1.463	0.462
12	1.423	0.448
13	1.146	0.710
14	1.195	0.538
15	1.162	0.676
`16	1.538	0.500
17	1.423	0.595
18	1.321	0.606
19	1.662	0.536
20	1.342	0.506

Table 3

The test for significant difference between the standard deviation measures and the geometric measure of variation showed that there was a significant difference between the geometric measures estimates and the standard deviation measure (t=19.5459, df=19, P-value<0.001). the test showed that the average estimates for the geometric measure of variation from the mean were significantly smaller than those of standard deviation (P-value<0.001).

The test results for efficiency of the geometric measure in comparison to standard deviation, based on the four measures of efficiency were as illustrated in table 4;

Measure of Efficiency	**Standard Deviation**	**Geometric Measure of Variation**

Coefficient of Variation	0.0970	0.2026
Relative Efficiency	1.540	
Bias	-0.006	0.042
Mean Squared Error	0.017	0.013

Table 4

Based on the coefficient of variation and Bias standard deviation is considered as the most efficient measure of variation about the mean, however, checking at the mean squared error and relative efficiency, the geometric measure of variation from the mean is the most efficient measure of variation about the mean. Standard deviation has a smaller coefficient of variation and a smaller bias, this shows that the estimates given by standard deviation are both closer to each other and closer to the parameter estimate, while for the case of the geometric measure of variation about the mean despite the estimates not being closer to each other or the true parameter, the variation within the geometric measure of variation is smaller than the variation within standard deviation, this shows that standard deviation is a more unbiased estimation technique for the geometric distributed datasets but geometric measure of variation about the mean is a more efficient measure of variation about the mean for large geometric datasets.

Small Skewed Continuous Datasets

Past studies have shown that standard deviation is affected by skewed datasets, as a result if a new measure of variation is to be developed, then it should be one that is not affected by skewness of data. In order to assess the efficiency of the geometric measures of variation in comparison to standard deviation in estimating the variation from the mean for small skewed samples, twenty samples each of size 10 from a chi-square distribution with 1 degree of freedom were simulated and assessed. The results of standard deviation and the geometric measure of variation from the mean for the samples were as illustrated in table 5;

Sample Number	**Standard Deviation**	**Geometric measure of variation**
1	1.102	0.482
2	1.242	0.587
3	1.718	0.899
4	0.551	0.323
5	1.127	0.554
6	1.724	0.887
7	2.342	1.463
8	1.394	0.769
9	1.532	0.910
10	1.136	0.508
11	3.244	1.653
12	1.325	0.340
13	1.558	0.741
14	1.380	1.049
15	0.815	0.572
`16	0.700	0.562
17	1.156	0.865
18	1.274	0.671
19	0.285	0.219
20	1.083	0.571

Table 5

The test for significant difference between the standard deviation measures and the geometric measure of variation showed that there was a significant difference between the geometric measures estimates and the standard deviation measure (t=7.744, df=19, P-value<0.001). the test showed that the average estimates for the geometric measure of variation from the mean were significantly smaller than those of standard deviation (P-value<0.001).

The test results for efficiency of the geometric measure in comparison to standard deviation, based on the four measures of efficiency were as illustrated in table 6;

Measure of Efficiency	Standard Deviation	Geometric Measure of Variation
Coefficient of Variation	0.4766	0.4880
Relative Efficiency	3.175	
Bias	0.149	0.006
Mean Squared Error	0.406	0.121

Table 6

Checking at the coefficient of variation alone, standard deviation is the most efficient measure of variation from the mean for skewed data sets because its standard deviation is relatively smaller than the mean compared to the geometric measure of variation from the mean, however, checking at the other three measures of efficiency, geometric measure of variation is the most efficient measure because it has smaller variance (relative efficiency), it has smaller bias compared to standard deviation and lastly it also have a smaller Mean Squared Error compared to standard deviation. Therefore, based on the analysis it can be concluded that geometric measure of variation is a more efficient measure of variation from the mean for small skewed continuous samples.

Large Skewed Continuous Datasets

Considering large continuous skewed samples, the assessment on the efficiency of geometric measure of variation from the mean in estimating the average deviation from the mean in comparison to standard deviation. Twenty samples each of size 200 from a chi-square distribution with 1 degree of freedom were simulated and assessed. The results of standard deviation and the geometric measure of variation from the mean for the samples were as illustrated in table 7;

Sample Number	Standard Deviation	Geometric Measure of Variation
1	1.548	0.605
2	1.330	0.611
3	1.196	0.559
4	1.360	0.608
5	1.415	0.677
6	1.298	0.578
7	1.333	0.632
8	1.442	0.627
9	1.350	0.664
10	1.855	0.757
11	1.249	0.629
12	1.265	0.627
13	1.248	0.632
14	1.527	0.706
15	1.313	0.537
`16	1.260	0.554
17	1.798	0.724
18	1.314	0.621
19	1.338	0.498
20	1.414	0.670

Table 7

The test for significant difference between the standard deviation measures and the geometric measure of variation showed that there was a significant difference between the geometric measures estimates and the standard deviation measure (t=25.292, df=19, P-value<0.001). the test showed that the average estimates for the geometric measure of variation from the mean were significantly smaller than those of standard deviation (P-value<0.001).

The test results for efficiency of the geometric measure in comparison to standard deviation, based on the four measures of efficiency were as illustrated in table 8;

Measure of Efficiency	**Standard Deviation**	**Geometric Measure of Variation**
Coefficient of Variation	0.1247	0.1014
Relative Efficiency	7.491	
Bias	0.010	0.003
Mean Squared Error	7.844	1.578

Table 8

The results in the above table illustrate that Geometric Measure of variation from the mean was more efficient than standard deviation. Geometric Measure of variation had a smaller coefficient of variation meaning that the estimates were close to each other (relatively smaller standard deviation compared to the mean), the variance of Geometric Measure of variation was smaller than that of standard deviation, this makes Geometric Measure of variation to have a smaller relative efficiency than that of standard deviation. In terms of bias, Geometric Measure of variation had a smaller bias compared to the standard deviation. Lastly, the mean squared errors for Geometric Measure of variation was also smaller than that of standard deviation. Therefore, for large continuous skewed samples, Geometric Measure of variation is the most efficient measure of variation from the mean compared to standard deviation.

Small Peaked Continuous Samples

Most peaked datasets are known to be prone to outliers because most of the observations are clustered at the middle with very few observations being observed at the extremes. Past studies have also determined that standard deviation is always affected outliers in the datasets. Hence a new measure of variation should not be affected by outliers. In order to assess the efficiency of the geometric measures of variation in comparison to standard deviation in estimating the variation from the mean for small peaked samples, twenty samples each of size 10 from a t-distribution with 1 degree of freedom were simulated and assessed. The results of standard deviation and the geometric measure of variation from the mean for the samples were as illustrated in table 9;

Sample Number	**Standard Deviation**	**Geometric Measure of Variation**
1	5.737	1.779
2	8.483	2.535
3	3.614	1.483
4	5.542	2.053
5	3.598	1.388
6	3.450	1.842
7	4.584	0.976
8	6.904	1.264
9	1.625	1.020
10	4.758	1.051
11	10.302	2.842
12	1.478	0.926
13	8.990	4.707
14	1.856	0.653
15	2.922	1.403
`16	0.591	0.376

17	6.291	2.104
18	5.593	1.802
19	1.480	0.311
20	1.247	0.599

Table 9

The test for significant difference between the standard deviation measures and the geometric measure of variation showed that there was a significant difference between the geometric measures estimates and the standard deviation measure (t=6.387, df=19, P-value<0.001). the test showed that the average estimates for the geometric measure of variation from the mean were significantly smaller than those of standard deviation (P-value<0.001).

The test results for efficiency of the geometric measure in comparison to standard deviation, based on the four measures of efficiency were as illustrated in table 10;

Measure of Efficiency	**Standard Deviation**	**Geometric Measure of Variation**
Coefficient of Variation	0.625	0.6489
Relative Efficiency	7.593	
Bias	0.659	0.595
Mean Squared Error	7.788	1.323

Table 10

Checking at the coefficient of variation alone, standard deviation is the most efficient measure of variation from the mean for peaked datasets because its standard deviation is relatively smaller than the mean compared to the geometric measure of variation from the mean, however, checking at the other three measures of efficiency, geometric measure of variation is the most efficient measure because it has smaller variance (relative efficiency), it has smaller bias compared to standard deviation and lastly it also have a smaller Mean Squared Error compared to standard deviation. Therefore, based on the analysis it can be concluded that geometric measure of variation is a more efficient measure of variation from the mean for small peaked continuous samples.

Large Peaked Continuous Samples

Considering large continuous peaked samples, the assessment on the efficiency of geometric measure of variation from the mean in estimating the average deviation from the mean in comparison to standard deviation. Twenty samples each of size 200 from a t-distribution with 1 degree of freedom were simulated and assessed. The results of standard deviation and the geometric measure of variation from the mean for the samples were as illustrated in table 11;

Sample Number	**Standard Deviation**	**Geometric Measure of Variation**
1	19.847	2.127
2	15.192	1.558
3	6.501	0.869
4	69.818	4.560
5	8.195	0.957
6	7.682	0.987
7	20.534	0.892
8	7.972	1.118
9	36.581	3.241
10	23.838	1.784
11	20.142	1.539
12	13.341	1.036
13	227.637	13.728
14	4.590	1.109

15	8.292	0.982
`16	10.485	2.127
17	53.748	3.677
18	57.052	3.872
19	20.123	1.713
20	33.229	3.661

Table 11

The test for significant difference between the standard deviation measures and the geometric measure of variation showed that there was a significant difference between the geometric measures estimates and the standard deviation measure (t=2.9501, df=19, P-value=0.004). the test showed that the average estimates for the geometric measure of variation from the mean were significantly smaller than those of standard deviation (P-value=0.008).

The test results for efficiency of the geometric measure in comparison to standard deviation, based on the four measures of efficiency were as illustrated in table 12;

Measure of Efficiency	**Standard Deviation**	**Geometric Measure of Variation**
Coefficient of Variation	1.4836	1.1148
Relative Efficiency	294.762	
Bias	25.166	1.478
Mean Squared Error	2944.155	10.023

Table 12

The results in the above table illustrate that Geometric Measure of variation from the mean was more efficient than standard deviation. Geometric Measure of variation had a smaller coefficient of variation meaning that the estimates were close to each other (relatively smaller standard deviation compared to the mean), the variance of Geometric Measure of variation was smaller than that of standard deviation, this makes Geometric Measure of variation to have a smaller relative efficiency than that of standard deviation. In terms of bias, Geometric Measure of variation had a smaller bias compared to the standard deviation. Lastly, the mean squared errors for Geometric Measure of variation was also smaller than that of standard deviation. Therefore, for large continuous peaked samples, Geometric Measure of variation is the most efficient measure of variation from the mean compared to standard deviation.

Conclusion

In conclusion, based on the findings made in the study, it can be concluded that geometric measure of variation from the mean do give smaller estimates from the mean than estimates from standard deviation. In terms of efficiency, the geometric measure of variation from the mean is the most efficient measure of variation from the mean for geometric, skewed and peaked datasets, this is because geometric and peaked datasets are very much prone to outliers a factor that interferes with the estimation of standard deviation. It is also efficient than standard deviation in estimating the average deviation from the mean for skewed datasets, this is because standard deviation assumes normality of data during estimation. Therefore, the geometric measure of variation about the mean is capable of solving the problems of standard deviation which is outlier and skewed effect of datasets.

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