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Performance Evaluation of Wound Rotor (Slip Ring) Induction Motor with Leading Power Factor

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Abstract: This paper presents an analysis of wound rotor induction motor with leading power factor. If rotor resistance to product of slip and rotor reactance (r_r/sx_r) ratio of an induction motor takes negative values, implying s being traditionally negative (super synchronous speed of the rotor), the rotor phasor current I_r, will lag the applied voltage by an angle greater than 90^o . This means negative power factor ($Cos\Phi_2$) or electric power flows out of the machine from rotor to the stator resulting in generation operation. In this paper, operation of an induction motor with a leading power factor is obtained by injecting capacitance, x_c whose magnitude is approximately twice that of the leakage reactance x_r say, the overall reactance of the rotor circuit becomes $-jx_r$ even though s is positive. The effective leakage reactance is capacitive such that power factor of the machine becomes leading instead of the usual inductive lagging power factor which will lead to electromagnetic torque in the opposite direction to the rotation of the rotor which is generation operation

Key words: Wound rotor induction motor; Leading power factor; Inverted torque-speed characteristics; Current loci; Induction generator.

1.1 Introduction

The field of application of alternating current (a.c) motors has increased considerably with the adoption of a.c system of distribution of electric energy for light and power. The induction motor is used in a wide variety of applications as a means of converting electric power to mechanical work. It is without doubt the workhorse of electric power industry [1-2]. The reasons are its low cost, simple and rugged construction, absence of commutator and good operating characteristics.

Induction motor can be classified as squirrel cage or wound rotor induction motor. The squirrel cage rotor winding are perfectly symmetrical and have advantage of being adaptable to any number of pole pairs. About 90 percent of induction motors are squirrel cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible Research has shown that the distribution of current due to electromagnetic induction in the rotor bars varies from bar to bar sinusoidally and depends upon the position and time, assuming sinusoidal distribution of radial flux density in space and also the applied voltage to be varying sinusoidally with time[3-4]. There is no possibility of adding any external resistance in the rotor circuit since the rotor winding is permanently short-circuited in cage construction.

In wound rotor induction motor, the rotor is wound with an insulated winding similar to the stator except that the number of slots is smaller and fewer turns per phase of a heavier conductor are used. Some machines are provided with brush lifting and slip-ring short-circuiting arrangement for running condition. Since the connection of the wound secondary to the external terminals is made through slip-rings and brushes, so wound secondary motors often are called slip-ring induction motors.

In its normal working range, the speed of the induction motor remains reasonably constant, varying slightly with load. For this reason it is regarded as a constant speed motor; the major shortcoming of the induction motor is the relative low power factor which is always lagging.

Another distinguishing feature of such motors is that they are singly excited machines, although such machines are equipped with both field and armature windings. In such a machine the field (or stator) winding is connected to an ac supply and there is no electrical connection from the armature (or rotor) to any source of supply. Currents are made to flow in the armature (or rotor) conductors by induction which interact with the field produced by the field (or stator) winding and thereby produce a net unidirectional torque. Such motors are also called asynchronous motors as they run at a speed other than the synchronous speed of rotating field developed by the stator currents.

Like other electrical machines, the asynchronous machine is reversible, i.e it can operate as both a motor and a generator. When run faster than its synchronous speed, an induction motor runs as an induction generator. It converts the mechanical energy it receives into electrical energy and this energy is released by the stator. However induction motor can be reversed without running it faster than synchronous speed by connecting capacitor which makes it operate with leading power factor[5].

2.1 Dynamic Model of an Induction Motor in Arbitrary Reference Frame.

Stator Voltage Equation

$$\left.\begin{aligned} V_{qs} &= r_s\, i_{qs} + w\lambda_{ds} + p\lambda_{qs} \\ V_{ds} &= r_s\, i_{qs} - w\lambda_{qs} + p\lambda_{ds} \\ V_{os} &= r_s\, i_{os} + p\lambda_{os} \end{aligned}\right\} \quad (1)$$

Rotor Voltage Equation

$$\left.\begin{aligned} V'_{qr} &= r'_r\, i'_{qr} + (w - w_r)\lambda'_{dr} + p\lambda'_{qr} \\ V'_{dr} &= r'_r\, i'_{dr} - (w - w_r)\lambda'_{qr} + p\lambda'_{dr} \\ V'_{or} &= r'_r\, i'_{or} + p\lambda'_{or} \end{aligned}\right\} \quad (2)$$

Flux Linkage

$$\left.\begin{aligned} \lambda_{qs} &= (L_{ls} + L_m) i_{qs} + L_m\, i_{qr} \\ &= L_{ls}\, i_{qs} + L_m\, (i_{qs} + i_{qr}) \\ \lambda_{ds} &= (L_{ls} + L_m) i_{ds} + L_m\, i_{dr} \\ &= L_{ls}\, i_{ds} + L_m \left(i_{ds} + i'_{dr} \right) \\ \lambda'_{qr} &= L'_{lr}\, i'_{qr} + L_m\, (i_{qs} + i'_{qr}) \\ \lambda'_{dr} &= L_{lr}\, i'_{dr} + L_m\, (i_{ds} + i_{dr}) \\ \lambda'_{or} &= L_{lr}\, i'_{or} \end{aligned}\right\} \quad (3)$$

Substituting the values of flux from equation (3) into the rotor and stator voltage equations (1) and (2),

$$\left.\begin{aligned} V_{qs} &= r_s\, i_{qs} + w\lambda_{ds} + p(L_{ls}\, i_{qs} + L_m\, (i_{qs} + l'_{qr}) \\ V_{qr}' &= r'_r\, i'_{qr} + (w - w_r)\lambda'_{dr} + p[L_{lr}\, i_{qr} + L_m\, (i_{qs} + i'_{qr}) \\ V_{ds} &= r_s\, i_{ds} - w\lambda_{qs} + p[(L_{ls} + L_m) i_{ds} + L_m\, i_{dr}] \\ V'_{dr} &= r'_r\, i'_{dr} - (w - w_r)\lambda'_{qr} + p[L_{lr}\, i'_{dr} + L_m\, (i_{ds} + i'_{dr}) \\ V_{os} &= r_s\, i_{os} + p\,(L_s\, i_{os}) \\ V'_{or} &= r'_r\, i'_{or} + p(L_{lr}\, i'_{or}) \end{aligned}\right\} \quad (4)$$

2.2 Induction Machine Equation in Stationary Reference Frame

By setting speed, w=0 in equation (1) and (2) gives:

$$\left.\begin{aligned} V_{qs}^{s} &= r_s\, i_{qs} + p\lambda_{qs} \\ V_{ds}^{s} &= r_s\, i_{ds} + p\lambda_{ds} \\ V_{os}^{s} &= r_s\, i_{os} + p\lambda_{os} \end{aligned}\right\} \quad (5)$$

Therefore, stator voltage equation,

$$V_{qs}^{s} = r_s\, i_{qs} + p\frac{w_b}{w_b}\lambda_{qs}$$

$$\Rightarrow \; V_{qs}^{s} = r_s\, i_{qs} + \frac{p}{w_b}\psi_{qs}$$

$$Where\ \psi = w_b\lambda$$

$$\left.\begin{aligned} V_{qs}^{s} &= \frac{p}{w_b}\psi_{qs} + r_s\, i_{qs} \\ V_{ds}^{s} & \frac{p}{w_b}\psi_{s} + r_s\, i_{ds}^{s} \\ V_{os} &= \frac{p}{w_b}\psi_{os} + r_s\, i_{os} \end{aligned}\right\} \quad (6)$$

Rotor Voltage Equations.

$$\left.\begin{aligned} V_{qr}^{'s} &= \frac{p}{w_b}\psi_{qr}^{s} - \frac{w_r}{w_b}\psi_{dr}^{'s} + r_r^{'}\, i_{dr}^{'s} \\ V_{dr}^{'s} &= \frac{p}{w_b}\psi_{qr}^{'s} + \frac{w_r}{w_b}\psi_{qr}^{'s} + r_r^{'}\, i_{dr}^{'s} \\ V_{or} &= \frac{p}{w_b}\psi_{or} + r_r^{'}\, i_{or}^{'} \end{aligned}\right\} \quad (7)$$

Flux Linkage Equations.

$$\begin{bmatrix} \psi_{qs}^{s} \\ \psi_{ds}^{s} \\ \psi_{os}^{s} \\ \psi_{qr}^{'s} \\ \psi_{dr}^{'s} \\ \psi_{or}^{'s} \end{bmatrix} = \begin{bmatrix} X_{ls}+X_m & 0 & 0 & X_m & 0 & 0 \\ 0 & X_{ls}+X_m & 0 & 0 & X_m & 0 \\ 0 & 0 & X_{ls} & 0 & 0 & 0 \\ X_m & 0 & 0 & X_{Lr}^{'}+X_m & 0 & 0 \\ 0 & X_m & 0 & 0 & X_{Lr}^{'}+X_m & 0 \\ 0 & 0 & 0 & 0 & 0 & X_{Lr}^{'} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \\ i_{0s}^{s} \\ i_{qr}^{'s} \\ i_{dr}^{'s} \\ i_{or}^{'} \end{bmatrix} \quad (8)$$

Torque equations,

$$\begin{aligned} Tem &= \frac{3}{2}\frac{P}{2w_b}\left(\psi_{dr}^{'s} - \psi_{dr}^{'s}\, i_{qr}^{'s}\right) \\ &= \frac{3}{2}\frac{P}{2w_b}\left(\psi_{ds}^{s}\, i_{qs}^{s} - \psi_{qs}\, i_{ds}^{s}\right) \\ &= \frac{3}{2}\frac{P}{2w_b}\, x_m\left(i_{dr}^{'s}\, i_{qs} - i_{qr}^{'s}\, i_{ds}^{s}\right) N.m \qquad (9) \end{aligned}$$

1.2 Equations for Simulation of Induction Motor in Stationary Reference Frame

The model equations 6, 7 and 8 may be rearranged into the following form for simulation:

Voltage and flux linkage equations are:

$$\left.\begin{aligned} \Psi_{qs}^{s} &= w_b \int (V_{qs}^{s} + \frac{r_s}{x_{LS}} (\Psi_{mq}^{s} - \Psi_{qs}^{s}))\, dt \\ \Psi_{ds}^{s} &= w_b \int (V_{ds}^{s} + \frac{r_s}{x_{LS}} (\Psi_{md}^{s} - \Psi_{ds}^{s}))\, dt \\ i_{os} &= \frac{w_b}{x_{LS}} \int (V_{os} - l_{os}\, r_s)\, dt \end{aligned}\right] \qquad (10)$$

$$\left.\begin{aligned} \Psi_{qr}^{s} &= w_b \int (V_{qr}^{'s} + \frac{w_r}{w_b} \Psi_{dr}^{'s} + \frac{r_r^{'}}{x_{Lr}^{'}} (\Psi_{mq}^{s} - \Psi_{qr}^{s}))\, dt \\ \Psi_{dr}^{'s} &= w_b \int (V_{ds}^{'s} + \frac{w_r}{w_b} \Psi_{dr}^{'s} + \frac{r_r^{'}}{x_{Lr}^{'}} (\Psi_{md}^{s} - \Psi_{dr}^{'s}))\, dt \\ i_{or} &= \frac{w_b}{x_{Lr}^{'}} \int (V_{or}^{'} - i_{or}\, r_r^{'})\, dt \end{aligned}\right] \qquad (11)$$

$$\left.\begin{aligned} \Psi_{mq}^{s} &= x_m\, (i_{qs}^{'s} + i_{qr}^{'s}) \\ \Psi_{mq}^{s} &= x_m\, (i_{ds}^{'s} + i_{dr}^{'s}) \end{aligned}\right] \qquad (12)$$

$$\left.\begin{aligned} \Psi^{s}{}_{qs} &= x_{Ls}\, i_{qs}^{'} + \psi_{mq}^{s}, & i_{qs}^{'s} &= \frac{\Psi_{qs}^{s} - \psi_{mq}^{s}}{x_{Ls}} \\ \Psi_{ds}^{s} &= x_{Ls}\, i_{ds}^{s} + \psi_{md}^{s}, & i_{ds}^{'s} &= \frac{\Psi_{qs}^{s} - \psi_{md}^{s}}{x_{Ls}} \\ \Psi_{qr}^{'s} &= \left(x_{Lr}^{'}\right) i_{qr}^{s} + \psi_{mq}^{s}, & i_{qr}^{'s} &= \frac{\Psi_{qr}^{'s} - \psi_{mq}^{s}}{(x_{Lr}^{'})} \\ \Psi_{dr}^{'s} &= \left(x_{Lr}^{'}\right) i_{dr}^{'s} + \Psi_{dr}^{'s}, & i_{dr}^{'s} &= \frac{\Psi_{dr}^{'s} - \psi_{md}^{s}}{(x_{Lr}^{'})} \end{aligned}\right] \qquad (13)$$

Where

$$\frac{1}{x_M} = \frac{1}{x_m} + \frac{1}{x_{ls}} + \frac{1}{x_{lr}^{'}} \qquad (14)$$

and

$$\begin{aligned} \psi_{mq}^{s} &= x_M \left(\frac{\psi_{qs}^{s}}{x_{ls}} + \frac{\psi_{dr}^{'s}}{x_{lr}^{'}} \right) \\ \psi_{md}^{s} &= x_M \left(\frac{\psi_{ds}^{s}}{x_{ls}} + \frac{\psi_{dr}^{'s}}{x_{lr}^{'}} \right) \end{aligned} \qquad (15)$$

The torque equation is

$$T_{em} = \frac{3}{2} \frac{P}{2w_b} (\Psi_{qs}^{s}\, i_{ds}^{s} - \Psi_{qs}^{s}\, i_{ds}^{s}) \quad N.m$$

The equation of motion of the rotor is obtained by equating the inertia torque to the accelerating torque, that is,

$$\frac{dw_{rm}}{dt} = T_{em} + T_{mech} - T_{damp} \qquad N.m \qquad (16)$$

Where

T_{mech} = the externally – applied mechanical torque in the direction of the rotor speed.
T_{damp} = the damping torque in the direction opposite to rotation.
The per unit speed, w_r/w_b needed for
Building the speed voltage terms in the rotor voltage equations, can be obtained by integrating

$$\frac{2Jw_b}{P}\frac{d(w_r/w_b)}{dt} = Tem + Tmech - Tdamp \quad N.m \quad (17)$$

Equation (16) can be written in terms of the inertia constant, H, defined as the ratio of the kinetic energy of the rotating mass at base speed to the rated power, that is

$$H = \frac{Jw_{bm}^2}{2S_b} \quad in \;\; per.unit \qquad (18)$$

Expressed in per unit values of the machine's own base power and voltage, equation (16) can be rewritten as $2H\frac{d(w_r/w_b)}{dt} = T_{em} + T_{mech} - T_{damp}$ *in per unit* (19)

The equation for transforming abc phase voltage to qd0 voltage is given as:

$$v_{qs}^s = 2/3v_{as} - 1/3v_{bs} - 1/3v_{cs}$$

$$= 2/3v_{ag} - 1/3v_{bg} - 1/3v_{cg} \qquad (20)$$

$$v_{ds}^s = 1/\sqrt{3}(v_{cs} - v_{bs}) = 1/\sqrt{3}(v_{ag} - v_{bg}) \qquad (21)$$

$$v_{cs} = 1/3(v_{as} + v_{bs} + v_{cs}) = 1/3(v_{ag} + v_{bg} + v_{cg} \quad (22))$$

The equation for transforming of qd0 stator current back to phase current is given as:

$$i_{as} = i_{qs} + i_{0s} \qquad (23)$$

$$i_{bs} = -1/2i_{qs} - \sqrt{3}/2i_{ds} + i_{0s} \qquad (24)$$

$$i_{cs} = -1/2i_{qs} - \sqrt{3}/2i_{ds} + i_{0s} \qquad (25)$$

The balanced supply voltage can be expressed as

$$\left.\begin{aligned} v_{as} &= v_{ms}\cos(w_e t) \\ v_{bs} &= v_{ms}\cos(w_e t - 2\pi/3) \\ v_{cs} &= v_{ms}\cos(w_e t - 4\pi/3) \end{aligned}\right\} \qquad (26)$$

2.3 Model Equations of Capacitive Induction Motor.

The transformations like dq developed by park can facilitate the computation of the transient solution of the induction machine model by transforming the differential equations with time-varying inductances to differential equations with constant inductances[7-11].

The qdo transformation matrix, $\lfloor T_{qdo}(\theta) \rfloor$ of stator of induction machine is

$$T_{qdo}(\theta)=\frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta-{2\pi}/{3}\right) & \cos\left(\theta+{2\pi}/{3}\right) \\ \sin\theta & \sin\left(\theta-{2\pi}/{3}\right) & \sin\left(\theta+{2\pi}/{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (27)$$

And its inverse is

$$\left[T_{qdo}\right]^{-1}=\begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta-{2\pi}/{3}\right) & \sin\left(\theta-{2\pi}/{3}\right) & 1 \\ \cos\left(\theta+{2\pi}/{3}\right) & \sin\left(\theta+{2\pi}/{3}\right) & 1 \end{bmatrix} \quad (28)$$

The transformation angle to the rotor phase quantities is (Θ - Θ_r). Therefore T (Θ - Θ_r) is the transformation angle.

$$T(\theta-\theta_r)=\frac{2}{3}\begin{bmatrix} \cos(\theta-\theta_r) & \cos\left(\theta-\theta_r-\frac{2\pi}{3}\right) & \cos\left(\theta-\theta_r+\frac{2\pi}{3}\right) \\ \sin(\theta-\theta_r) & \sin\left(\theta-\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta-\theta_r+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (29)$$

$$T(\theta-\theta_r)^{-1}=\begin{bmatrix} \cos(\theta-\theta_r) & \sin(\theta-\theta_r) & 1 \\ \cos\left(\theta-\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta-\theta_r-\frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta-\theta_r+\frac{2\pi}{3}\right) & \sin\left(\theta-\theta_r+\frac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (30)$$

For stator

$$i_{abcs}=pq_{abcs} \quad (31)$$

where $p=\frac{d}{dt}$

q = quantity of charge.

Applying Park's transformation matrix equation (27) and (28) to equation (31):

$$i_{qd0s}=T_{qd0}(\theta)p\left[\left(T_{qd0}(\theta)\right)^{-1}\right]q_{qd0s} \quad (32a)$$

This gives

$$i_{qd0s}=T_{qd0}(\theta)p\left[\left(T_{qd0}(\theta)\right)^{-1}\right]q_{qd0}+T_{qd0}(\theta)\left(T_{qd0}(\theta)\right)^{-1}pq_{qd0s} \quad (32b)$$

$$i_{qd0s}=wq_{qd0s}+pq_{qd0s} \quad (33)$$

where

$$\left(q_{qd0s}\right)^T=\left[q_{ds} \quad -q_{qs} \quad 0\right] \quad (34)$$

In expanded form

$$i_{qs} = wq_{ds} + pq_{qs} \quad (35a)$$

$$i_{ds} = -wq_{qs} + pq_{ds} \quad (36a)$$

$$i_{0s} = pq_{0s} \quad (37a)$$

Substituting equation 21 in equation 18a

$$but \;\; q_{qd0} = cv_{qd0} \quad (38)$$

c is the capacitance

through 20a.

$$i_{qs} = wcv_{ds} + cpv_{qs} \quad (35b)$$

$$i_{ds} = -wcv_{qs} + cpv_{ds} \quad (36b)$$

From equation 35b

$$i_{0s} = cpv_{0s} \quad (37b)$$

$$cpv_{qs} = i_{qs} - wcv_{ds}$$

$$v_{qs} = \int \left(\frac{i_{qs}}{c} - wv_{ds} \right) dt \quad (39)$$

From equation 36b

$$cpv_{ds} = i_{ds} + wcv_{qs}$$

$$\therefore$$

$$v_{ds} = \int \left(\frac{i_{ds}}{c} + wv_{qs} \right) dt \quad (40)$$

From equation 40b

$$v_{0s} = \int \frac{i_{0s}}{c} dt \quad (41)$$

For rotor

$$i_{abcr} = pq_{abcr} \quad (42)$$

The transformation angle to the rotor phase quantities is ($\theta - \theta_r$). Therefore

$T(\theta - \theta_r)$ is the transformation angle.

Applying the transformation matrix equation (29) and (30) to equation 42 in the same manner as in that of stator, the equation will be obtained as

$$i_{qdor} = (w - w_r)q_{qd0r} - pq_{qdor} \quad (43)$$

In expanded form,

$$i_{qr} = (w - w_r)cv_{dr} + cpv_{qr} \quad (44)$$

$$i_{dr} = (w - w_r)cv_{qr} - cpv_{dr} \quad (45)$$

$$i_{0r} = cpv_{0r} \quad (46)$$

From equation 43,

$$cpv_{qr} = i_{qr} - (w - w_r)cv_{dr}$$

$$v_{qr} = \int \left(\frac{i_{qr}}{c} - (w - w_r)v_{dr} \right) dt \quad (47)$$

From equation 45

$$cpv_{dr} = i_{dr} + (w - w_r)cv_{qr}$$

$$\therefore$$

$$v_{dr} = \int \left(\frac{i_{dr}}{c} + (w - w_r)v_{qr} \right) dt \quad (48)$$

From equation 46

$$v_{0r} = \int \frac{i_{0r}}{c} dt \qquad (49)$$

2.2 Steady State Analysis of Induction Motor

In a steady state operation, the derivative terms in the voltage equation are made zero.

From equation (1)

$$V_{qs} = r_s\, i'_{qs} + w\lambda_{ds} \qquad (50).$$

Similarly from equation (2),

$$V'_{qr} = r'_r\, i'_{qr} + (w - w_r)\lambda'_{dr} \qquad (51)$$

Using the transformation $f_{ds} = j\, f_{qs}$ (since q and d are in space quadrature).

Therefore,

$$\left.\begin{aligned} V_{qs} &= r_s\, i_{qs} + jw\lambda_{qs} \\ V'_{qr} &= r'_r\, i'_{qr} + j(w - w_r)\lambda'_{qr} \end{aligned}\right\} \qquad (52).$$

From equation (3) ,Flux Linkage

$$Also\ \lambda'_{qr} = L_m\, i_{qs} + (L'_{Lr} + L_m) i'_{qr}$$

$$= L_m\, i_{qs} + L'_{lr}\, i'_{qr} + L_m\, i'_{qr}$$

$$= L'_{Lr}\, i'_{qr} + L_m\left(i_{qs} + i'_{qr}\right) \qquad (53)$$

Now substituting (52) and (53) into equation (54) we have

$$V_{qs} = r_s\, i_{qs} + jw\left[L_{LS} i_{qs} + L_m\left(i_{qs} + i'_{qr}\right)\right]$$

$$= r_s\, i_{qs} + jw\, L_{Ls}\, i_{qs} + jwL_m\left(i_{qs} + i'_{qr}\right) \qquad (54)$$

$$Also$$

$$V_{qr} = r_s i'_{qr} + j(w - w_r)\lambda'_{qr}$$

$$= r'_r i_{qr} + jsw\lambda'_{qr},\ where\ s = Slip$$

$$= r'_r i'_{qr} + jsw\left[L'_{qr} i'_{qr} + L_m\left(i_{qs} + i_{qr}\right)\right] \qquad (55)$$

To obtain the steady-state equivalent circuit, it is assumed that q-axis is aligned with ‘a’ phase of the machine such that

$$V_{qs} = V_{as};\ i_{qs} = i_{as};$$

$$V'_{qr} = V'_{ar};\ i'_{qr} = i'_{ar}$$

Equation 57 becomes

$$V_{as} = r_s i_{as} + jx_{Ls} i_{as} + jx_m\left(i_{as} + i_{ar}\right) \qquad (56)$$

While equation 16 becomes

$$V'_{ar} = r'_r\, i'_{ar} + jsx_{lr}\, i'_{as} + jsx_m\left(i'_{ar} + i_{as}\right) \qquad (57)$$

Dividing both sides of equation (57) by s, we have $\frac{V_{ar}}{s} = \frac{r'_r}{s} i_{ar} + jx'_{lr}\, i'_{as} + jx_m\left(i_{as} + i'_{ar}\right) \qquad (58)$

Now the steady state equivalent circuit can be drawn using equation (56) and (58) as shown below:

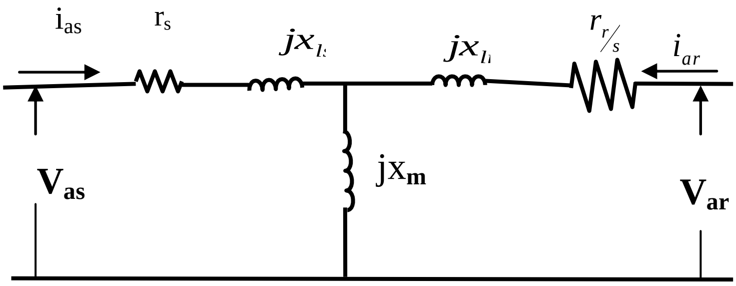

Figure 1: Per phase equivalent circuit of a poly-phase induction motor. V_{ar}/s

In induction machine, the rotor winding are normally short-circuit and thus no voltage is applied to the rotor. Hence $V_{ar} = 0$ and the equivalent circuit is as shown in figure 2.

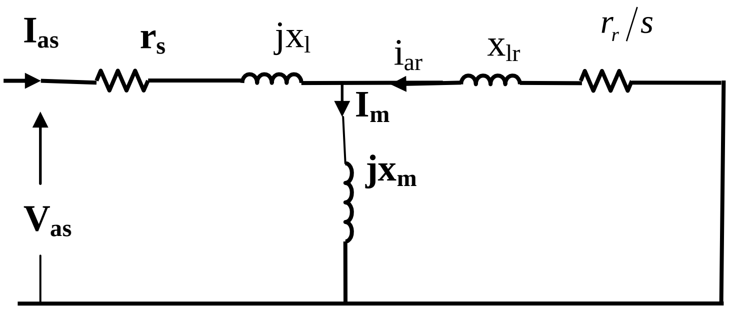

Figure 2: Per-phase equivalent circuit of a poly-phase induction motor with rotor short-circuited.

3.1 Current Loci of Induction Motors.

The rotor circuit voltage equation of an induction motor in phasor form can be written from its equivalent circuit of figure 3 [6] as

$$E_r = I_r\left(\frac{R_r}{s} + jX_r\right) \quad (59)$$

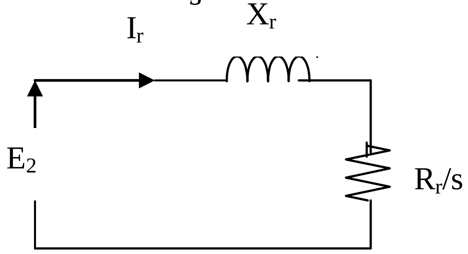

Figure 3: Rotor equivalent circuit of an induction motor.

From equation (59), the per-phase rotor current at any slip s is given by:

$$I_r = \frac{E_2\angle 0}{\sqrt{\left(\left(\frac{R_r}{s}\right)^2 + x_r^2\right)}\angle\theta_2} \quad (60)$$

Where $\theta_2 = Arc\tan\frac{sX_r}{R_r}$

And the rotor current lags the rotor voltage E_2 by θ_2. The power input to the rotor is given by:

$P_g = E_2 I_r \cos\theta_2$ (61)

$$\cos\theta_2 = \frac{per\ phase\, rotor\, resis\tan ce}{per\ phase\, rotor\, impedance}$$

The power factor $\cos\Theta_2$ may be expressed as; $= \dfrac{R_r/_s}{\sqrt{\left(\dfrac{R_r}{s}\right)^2 + X_r^2}}$ (62)

Substituting equation (62) into (61) gives:

$$P_g = E_2 I_r \frac{R_r/s}{\sqrt{(R_r/s)^2 + X_r^2}} = I_r^2\, R_r/s \qquad (63)$$

P_g is actually the power transformed from the stator to the rotor across the air gap. For the purpose of obtaining the rotor locus, (61) can be rewritten as: $\dfrac{-jE_2}{X_r} = I_r - jI_r \dfrac{R_r}{sX_r}$ (59*a*)

Equation (66a) shows that a constant current E_2/X_r lagging 90^0 behind E_2 is made up of two components: the current in the rotor circuit I_2 plus variable

component $I_r \dfrac{R_r}{sX_r}$ lagging I_2 by 90^0 as shown in figure 3.

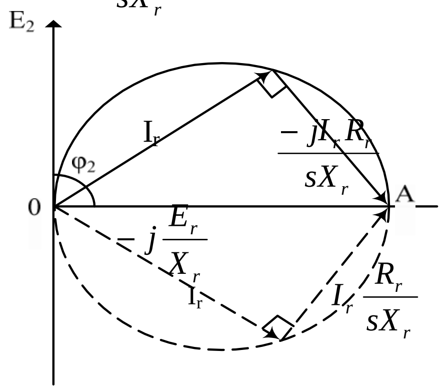

Figure 4: Current locus of inductive rotor circuit of induction motor.

From the geometry of the phasor diagram shown in fig 4, where a right-angled triangle is formed over a constant diameter E_2/X_r it can be seen that the phasor I_2 traces out semi-circle as $\dfrac{R_r}{sX_r}$ varies from 0 to ∞.

If $\dfrac{R_r}{sX_r}$ takes negative values, implying s being traditionally negative, (super synchronous speed of the rotor), equation 59a modifies to: $\dfrac{-jE_2}{X_r} = I_r + jI_r \dfrac{R_r}{sX_r}$ (59*b*)

The phasor I_2 will trace out semi-circle below the OA co-ordinate, which is the negative slip region. The phasor I_2 will now lag the applied voltage by an angle greater than 90^0. This means negative power factor ($\cos\Theta_2$) or that electric power flows out of the machine from rotor to the stator resulting in generator

Operation $\left(P_g = -I_r^2\, R_r/s\right), a$ reversal of power flow. Suppose capacitance is injected into the rotor circuit through the slip-rings and of magnitude twice that of the leakage reactance X_2 say, the overall reactance of the rotor circuit becomes–jX_2 even though s is positive. Equation 59a will now be modified to

$$j\frac{E_2}{X_r} = I_r + jI_r \frac{R_r}{sX_r} \qquad (59c)$$

From equation 59c), the locus of I_2 as s varies from zero to ∞ is shown in fig 4 below:

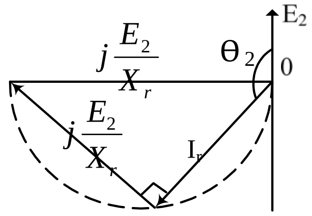

Figure 5: Current locus for the capacitive rotor circuit of induction machine.

It can be seen from figure 4 that the rotor current I_2 leads to the applied voltage E_2 by $\theta_2 > 90$. Consequently, the leading power factor Cos θ_2 is negative, which means that power flows out of the machine from the rotor to the stator resulting in generating operation $\left(P_g = -I_r^2 \frac{R_r}{sX_r}\right)$.Therefore, the generating operation can be obtained at rotor speed below synchronous ($w < w_s$) by capacitive injection into the rotor circuit, such that the overall reactance of the rotor circuit becomes capacitive (i.e. injected capacitive reactance X_c is greater than the leakage reactance X_r).

3.1 Performance Equations of an Induction Motor with Capacitor Injected at the Rotor.

If capacitor is injected into the rotor of an induction motor such that the reactance of capacitor (x_c) is more than the reactance of inductor, x_{lr} ($x_c > x_{lr}$), the torque can be derived from the equivalent circuit of figure 5 below.

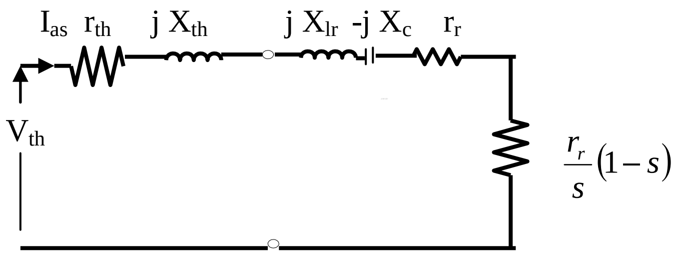

Figure 6:Thevenin equivalent's circuit of capacitive induction motor

$$\left.\begin{aligned} V_{th} &= \frac{jx_m}{r_s + j(x_{ls} + x_m)} X \; V_{as} \\ Z_{th} &= \frac{jx_m (r_s + jx_{ls})}{r_s + j(x_{ls} + x_m)} \end{aligned}\right\} \qquad (64) \qquad 3.74$$

where V_{th} and Z_{th} are Thevenin equivalent voltage and impedance respectively.

$$I_{aS} = \frac{v_{th}}{\left(r_{th} + r_{\frac{r}{s}}\right) + j(x_{th} + x_{lr} - x_c)} \qquad (65)$$

The mechanical power developed is given as

$$P = 3I_{as}^2 \frac{r_r}{s}(1-s) \qquad (66)$$

Substituting equation (68) into equation (69) gives

$$P = \frac{1}{w_r} x \frac{3V_{th}^2 \frac{r_r}{s}(1-s)}{\left(r_{th} + \frac{r_r}{s}\right)^2 + (x_{th} + x_{lr} - x_c)^2} \qquad (67)$$

The input power of the capacitive wound rotor induction motor is given by

$$P = 3V_s I_{as} \cos\varphi_2 \qquad (68)$$ $\cos\varphi_2$ = power factor.

For capacitive loaded motor, $\cos\varphi_2$ is negative, and hence the power input to the rotor is negative.

The developed torque is given by

$$T = \frac{-P}{w_r} = \frac{-P}{w_s(1-s)} \qquad (69)$$

Therefore substituting equation (67) in equation 69, the developed torque is,

$$T = \frac{1}{w_s} x \frac{-3V_{th}^2 \frac{r_r}{s}}{\left(r_{th} + r_{\frac{r}{s}}\right)^2 + (x_{th} + x_{lr} - x_c)^2} \qquad (70)$$

Table 3.1: A 4-pole wound Rotor induction motor parameters

V_{as}	240V
r_s	0.09 Ω
r_r	0.1 Ω
X_s	0.2 Ω
X_r	0.5 Ω
Xm	25 Ω
xc	0.8 Ω
F	50Hz
Rated power s_b	2781watts
P	4 poles
Moment of inertia, J_{rotor}	$0.1kgm^2$
Damping coefficient, b	0.9

The values of parameters in table 3.1 were obtained by constantly changing the values in matlab environment until appropriate torque speed curve is obtained as shown in figure 7.

3.2 Torque-Speed Characteristic of the 4-pole Wound rotor Induction Motor with Leading Power Factor.

The torque speed characteristic of the 4-pole wound rotor induction motor below (figure 7) is obtained by substituting the values of parameters in table 3.1 into equation 70

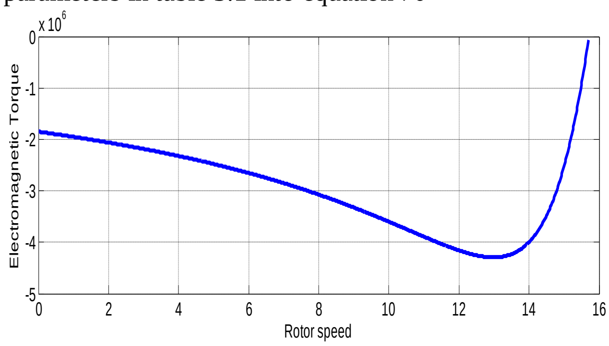

Figure 7: Inverted torque Speed characteristic of the 4-pole wound rotor induction motor

3.3 Simulation Result of the 4-pole wound rotor induction motor.

The parameters for simulation of the 4-pole wound rotor induction motor are shown in table 3.1:

The mechanical torque is the externally applied torque in the direction of the rotor speed.

The time and value arrays of the mechanical torque, T_{mech} repeating sequence source signal for simulation of the 4-pole wound rotor induction motor is shown below:

T_{mech_time}= [0 0.75 0.75 1.25 1.25 1.5 1.5 2]

$T_{mech_}$value = [0 0 -90 -90 -90 – 90 0 0]

The simulation results of the 4-pole wound rotor induction motor are shown below:

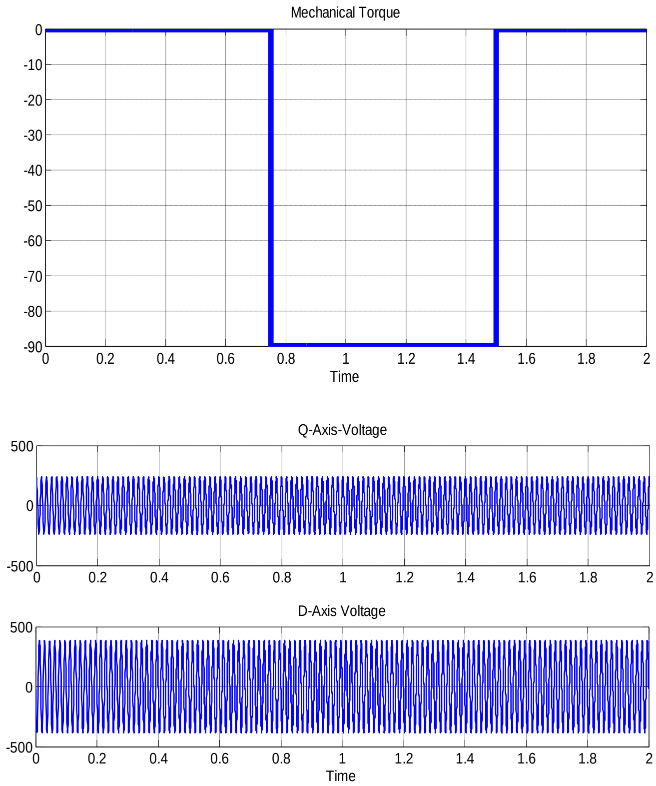

Figure 8: Simulation graph of the 4-pole wound rotor induction motor

4.1 Discussion of Result

If a capacitor whose reactance is twice that of the overall induction reactance is connected across the slip-rings of the 4-pole wound rotor induction motor such that the power factor is leading, the motor operates with inverted torque about the speed axis. Figure 6 shows the inverted torque-speed characteristics of an induction motor. The inverted torque was as a result negative power obtained since power factor is leading.

Using a stop time of 2 seconds, the 4-pole wound rotor induction motor was simulated starting from rest with rated voltage applied and with mechanical or load torque of 0 to 90N.m to 0. From the simulation result, as the mechanical torque changes there is a corresponding change in speed and electromagnetic torque. The d-axis voltage and q-axis voltage are sinusoidal. Since power is negative, the computed inertia constant is negative. As a result, the speed is inverted as shown in figure 7. Induction motor is motoring when mechanical torque is negative and generating when mechanical torque is positive.

4.2 Conclusion

Wound rotor induction motor with leading power factor has been analyzed. If capacitor connected to the rotor winding of the 4-pole wound rotor induction motor through slip ring is such that the reactance is capacitive (capacitive reactance is more than overall inductive reactance), the current in the winding will then be leading the voltage and hence the current direction will be reversed. This reversal of the current will lead to electromagnetic torque in the opposite direction to the rotation of the rotor, which is generation operation. The developed power becomes negative. Therefore, the generating operation can be obtained at rotor speed below synchronous ($w < w_s$) by capacitive injection into the rotor circuit, such that the overall reactance of the rotor circuit becomes capacitive.

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