

## Comparative Analysis of Different Control Algorithms Performances on a DC Servo Motor Position Control

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**Abstract:** The higher productivity demanded in most of the industrial applications places a tremendous pressure on the mechanisms of our electrical machines. However, the presence of nonlinearities poses the machines to deviate from its acceptable performance range. Therefore, the need to deploy a desirable control technique in order to achieve good tracking ability that would ultimately put the motor to operate at a desired set target is of paramount importance. The performance of the conventional controls on the system is unrealistic. This paper is aimed at comparing the various control algorithms performances of a direct current servo motor (DCSM) position control. The proposed controllers are the Model Predictive Controller (MPC), Proportional Controller (P), Proportional Integral (PI), Proportional Derivative (PD) and the Linear Quadratic Regulator (LQR). The controllers were investigated by the control response parameters viz: Rise Time, Settling Time and the Overshoot. Simulation results prove that the MPC could be a suitable controller of the position control of the DCSM.

**Keywords:** DC Servo motor, Model Predictive Controller (MPC), Proportional Controller (P), Proportional Integral (PI), Proportional Derivative (PD) and Linear Quadratic Regulator (LQR).

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### 1. Introduction

Designing a reliable control that would handle the issue of nonlinear effects which profusely affect the stable operation of our electric motors is quite critical. The higher demands on the dynamics of the electrical machines by many industrial applications fuel the need to furnish an appropriate control to maintain the desired stability.

However, there are certain kinds of application where rotation of the motor is needed for some particular angles not continuously for long period of time. Therefore, for this reason, some special types of electric motors having a special kind of arrangement that would make the motor to rotate a certain angle for a given input. For this reason servo motor comes into play. DCSM is a simple motor which is usually controlled for specific angular rotation with the aid of a special arrangement normally a closed loop feedback control system called SERVOMECHANISM. The DCSM has so many applications. Some of the applications are seen in remote controlled toy cars for controlling direction of motion and it is also used as the motor which moves the tray of a CD or DVD player. The main reason behind using a servo is that it provides angular precision, i.e. it will only rotate as much we want and then stop and wait for next signal to take further action. This is unlike a normal electrical motor which starts rotating as and when power is applied to it and the rotation continues until we switch off the power.[1]

However, literature study shows that several control strategies were proposed for the DCSM. In a study[2], the author proposed a conventional PID controller based on Particle Swarm optimization (PSO) algorithm for a DCSM. Of course the results obtained with the PID based on PSO should be better than the normal PID controller. The simulation results show that the proposed objective function provides more proficient in improving the dynamic response, better convergence, and fast response than the other methods based on maximum overshoot, rise time, and settling time of system step response. But in this paper, a complex controller, MPC, will be tested on the system to obtain better results. In another study[3], the authors proposed a new control scheme for the real-time distributed control on the Lon Works/IP VDN. It was based on IMC based on modified Smith predictor, model based disturbance observer, and feedback compensator for the cancellation of the high frequency noise and disturbance. The proposed control scheme proved to be robust even under the model mismatch and the prediction error in the time varying delay. The proposed control scheme is, therefore, expected to improve the quality and reliability of real-time distributed control of the process on the factory floor. Another authors proposed Fuzzy, Sliding Mode and Sliding Mode Fuzzy Controllers for the position control of the servo motor in [4]. In this work, Sliding mode fuzzy control is implemented. It was observed that the system performance increased when compared to PID and fuzzy control for parameter variation case and for disturbance case which indicated the robustness of SMFC. So sliding mode fuzzy control is superior when compared to other controller in terms of control performance. But result was found to be

ineffective with the fuzzy controller due to the disturbances issue when compared to SMFC. Therefore, tuning method such as Bacterial Foraging Optimization Algorithm (BFOA) could be tested to optimize the parameters of the fuzzy controller. Also, in a study [5], the authors proposed two control algorithms such as PID controller and predictive controller for a DC servo motor. Performances of these controllers were explored through simulation using MATLAB/SIMULINK software. According to the simulation results the Comparisons between PID (ZN), PID (GG) and MPC controller are given. The tuning method was more efficient in improving the step response characteristics such as, reducing the rise time, settling time and maximum overshoot in Position control of DC servo motor. Model predictive controller method gives the best performance and superiority of MPC method compare to other controller. The ZN method is conventional which is not suitable for obtaining optimal parameters. The BFOA and PSO could be used to prove the effectiveness of the proposed GG method.

## 2. DC Servo Motor Modeling

The DC servo motor is the one that can be controlled with the help of a feedback closed loop system known as servomechanism. The purpose of this system in the DCSM is to help in controlling the motor to certain angle of rotation. The motor can be controlled by two methods: I Armature control and II field control. The former is commonly used because of the faster response than in the later. The motor parameters are defined in Table I and the mathematical model of the motor can be obtained as in [5] by considering the Figure 1.

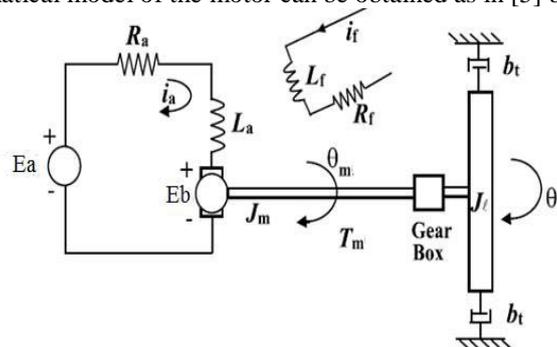


Figure 1: Armature Controlled DCSM (Source: [5])

From the Newton second law, we can obtain the torque as:

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \quad (1)$$

Similarly, using the KVL

$$e_a = L_a \frac{di_a}{dt} + R_a i_a + e_b \quad (2)$$

$$e_b = K_b \cdot \frac{d\theta}{dt} \quad (3)$$

The armature current and the torque are related by the Eqn:

$$T = K i_a \quad (4)$$

Substituting Eqn. (3) into Eqn. (2) and eliminating the armature current, and then re-arranging the equation and taking the Laplace transform we would arrive at:

$$\frac{\theta(s)}{E_a(s)} = \frac{K}{JL_a s^3 + (JR_a + BL_a) s^2 + (BR_a + KK_b) s} \quad (5)$$

Therefore, Eqn. (5) is the required mathematical model of the DCSM.

### 2.1 Model Predictive Controller

Model predictive controller is an advanced control algorithm which can predict error in a control system and provide the control actions appropriately. This paper applies this control scheme by using the mathematical model of the DC servo motor. However, the tuning parameters of the MPC such as the prediction horizon, P, defines the number of predictions needed to be taking to estimate the error, the control horizon, m, which defines the number of manipulated variable moves that is to be optimized at a given control interval. The control interval is the one that determines the sampling period T which is the control interval duration. In this work, the number of predictions was found to be 50 at a sampling period of 0.2 second. Normally, for this type of system the control horizon is 2.

However, the general description of the model predictive controller and its design could be understood using a simple SISO system describes in[6] as follows: The design objective of model predictive control is to compute a trajectory of a future manipulated variable  $u$  to optimize the future behavior of the plant output  $y$ . The optimization is performed within a limited time window by giving plant information at the start of the time window. Model predictive control systems are designed based on a mathematical model of the plant. The model to be used in the control system design is taken to be a state-space model. By using a state-space model, the current information required for predicting ahead is represented by the state variable at the current time. For simplicity, we begin our study by assuming that the underlying plant is a single-input and single-output system, described by:

$$x_m(k+1) = A_m x_m(k) + B_m u(k) \quad (6)$$

$$y(k) = C_m x_m(k) \quad (7)$$

Where,  $u$  is the manipulated variable or input variable;  $y$  is the process output; and  $x_m$  is the state variable vector with assumed dimension  $n$ . Note that this plant model has  $u(k)$  as its input. Thus, we need to change the model to suit our design purpose in which an integrator is embedded. Note that a general formulation of a state-space model has a direct term from the input signal  $u(k)$  to the output  $y(k)$  as:

$$y(k) = C_m x_m(k) + D_m u(k) \quad (8)$$

However, due to the principle of receding horizon control, where current information of the plant is required for prediction and control, we have implicitly assumed that the input  $u(k)$  cannot affect the output  $y(k)$  at the same time. Thus,  $D_m=0$  in the plant model. Taking a difference operation on both sides of Eqn. 6, we obtain that.

$$\begin{aligned} & x_m(k+1) - x_m(k) \\ = & A_m(x_m(k) - x_m(k-1)) + B_m(u(k) - u(k-1)) \end{aligned} \quad (9)$$

Let us denote the difference of the state variable by

$$\begin{aligned} \Delta x_m(k+1) &= x_m(k+1) - x_m(k); \\ \Delta x_m(k) &= x_m(k) - x_m(k-1) \end{aligned} \quad (10)$$

And the difference of the control variable by

$$\Delta u(k) = u(k) - u(k-1) \quad (11)$$

These are the increments of the variables  $x_m(k)$  and  $u(k)$ . With this transformation, the difference of the state-space equation is:

$$\Delta x_m(k+1) = A_m \Delta x_m(k) + B_m \Delta u(k) \quad (12)$$

Note that the input to the state-space model is  $\Delta u(k)$ . The next step is to connect  $\Delta x_m(k)$  to the output  $y(k)$ . To do so, a new state variable vector is chosen to be

$$X(k) = \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} \quad (13)$$

Where, superscript  $T$  indicates matrix transpose.

Note that

$$(k+1) - y(k) = C_m(x_m(k+1) - x_m(k)) = C_m \Delta x_m(k+1) = C_m A_m \Delta x_m(k) + \Delta u(k) \quad (14)$$

Putting together Eqn. (13) with Eqn. (14) leads to the following state-space model:

$$\begin{bmatrix} \Delta x_m(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} A_m & 0_m \\ C_m A_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) \quad (15)$$

$$y(k) = [0_m \ 1] \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} \quad (16)$$

More so, the manipulated input for a SISO system as describes in[7] as in Figure 2:

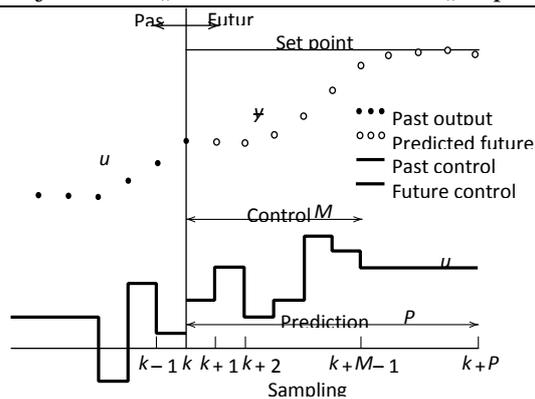


Figure2: Basic Concept of MPC

### 3. Controller Design

This section presents the description and the design of P, PI and PD controllers. The details of the description can be explained as in [8].

#### 3.1 Proportional Controller

In this type of controller, there is a linear relation between the output of the controller and actuating error signal as seen in Eqn. (17).

$$u(t) = k_p e(t) \quad (17)$$

Where,  $k_p$  is known as proportional gain,  $u$  is the output of the controller in this case.

The design of the proportional controller is illustrated in Figure 3. The controller P is connected in series with the plant and a unity feedback is connected. In this case, the difference between the actual output and the desired output is taken and the error signal is sent to the proportional controller and takes the control action appropriately. Then the control signal is sent to the plant and the new output is obtained. The proportional controller reduces the rise time, increases overshoot and there is small change in the settling time but does not eliminate the steady state error.

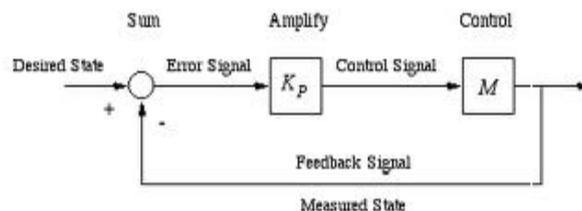


Figure 3:Proportional Controller (Source:[9])

#### 3.2 Integral Controller

In this controller, the output of the controller is changed at a rate which is proportional to the error signal. This controller can be designed when the proportional controller of figure 3 is replaced by the integral controller. The working principle remains the same but in this case, the controller is giving the integral of the actuating error signal. This controller eliminates the steady state error, decreases rise time, increases settling time and overshoot.

$$u(t) = k_i \int e(t) dt \quad (18)$$

Where,  $e(t)$  is the error,  $k_i$  is the integral controller gain.

#### 3.3 Proportional Integral Controller

A PI controller is obtained by combination of proportional and integral control action. Figure 4, illustrates the design of the PI controller. This controller would improve the steady state accuracy but increases rise time. This control action can be represented mathematically in Eqn. (19).

$$u(t) = k_p e(t) + k_i \int e(t) dt \quad (19)$$

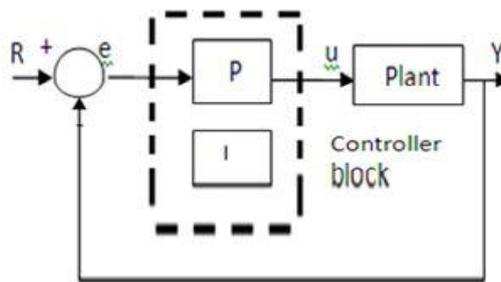


Figure 4: Proportional Integral Controller (Source: [9])

### 3.4 Proportional Derivative Controller

A PD controller is obtained by the combination of the proportional and derivative control action. This controller can be illustrated when the PI of Figure 4 is replaced with PD controller. The working principle of all this class of controller remains the same with only difference in the type of the control action. This type of controller reduces the rise time and the over shoot. It should be noted that in this paper, the controllers are evaluated by the commonly used transient response parameters such as the settling time the rise time and the over shoot. Therefore, the characteristics such as bandwidth, steady state response, damping etc. have not been talked about. Mathematically, this controller can be represented in Eqn. (20).

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} \quad (20)$$

### 3.5 Introduction to the Optimal Control

In many control situations, time and cost considerations force the control engineer to consider optimization of control efforts as well as response accuracy. The most effective and widely used design technique for linear systems is the LQR. The approach is to find the control input  $u$  which minimizes the quadratic cost function:

$$J = \int (x^T Q x + u^T R u) dt \quad (21)$$

Where, the weighing matrices are in Eqns. (22) and (23).

$$Q = Q^T > 0 \quad (22)$$

$$R = R^T > 0 \quad (23)$$

Therefore, an optimal control is one which is capable of driving the state to zero in order to obtain finite and constant  $J$  (i.e.  $x \rightarrow 0$  and  $u \rightarrow 0$ ) as in [10]. The LQR provides an optimal control law for a linear system with a quadratic performance index [11]. [12] States that a linear quadratic problem is probably the most celebrated optimal control problem. Similarly, [13] emphasized that the ultimate goal of optimal control is to minimize effort required or to maximize the desired benefit.

## 4. Results and Discussion

Simulation based on the mathematical model for the DC servo motorsystem by using MATLAB software has been performed. This would give a possibility of testing the proposed controllers which were evaluated by some commonly used transient response characteristics such as rise time ( $t_r$ ), settling time ( $t_s$ ), and overshoot. Table 1 shows the simulation parameters obtained from the data sheet. Figure 5 illustrates the response of the system when excited by a step input signal. However, Figures 6 to 10 shows the performance of MPC, P controller, PD controller, PI, combined P, PI and PD controllers and Figure 11 show the performance of the LQR controller.

**Table 1:** Simulation Parameters

PARAMETERS	VALUES
Moment of Inertia (J)	0.02kg.m <sup>2</sup>
Torque Constant (K <sub>t</sub> )	0.023Nm/A
Electromotive Force Constant (K <sub>b</sub> )	0.023Vs/rad
Linear Viscous Friction (B <sub>m</sub> )	0.03Nms/rad

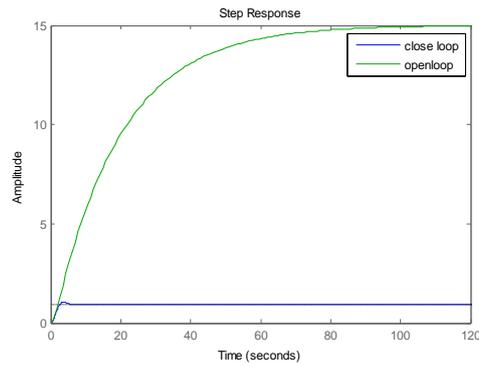


Figure 5: Step Response

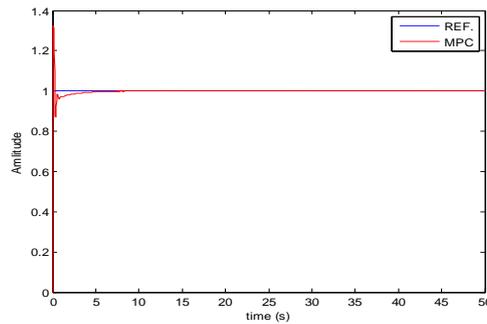


Figure6: Performance of MPC controller

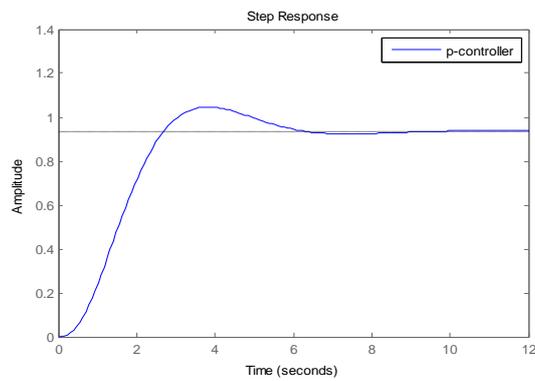


Figure7:Performance of P-Controller

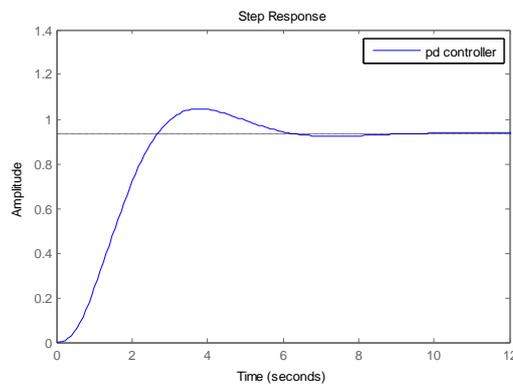


Figure 8: Performance of PD Controller

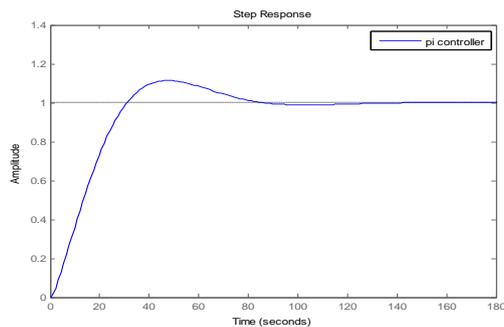


Figure 9: PI controller

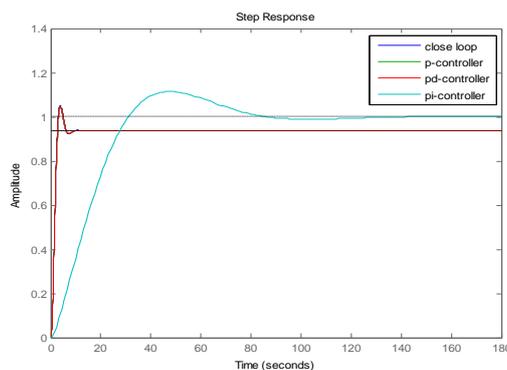


Figure 10: Performances of P, PD, and PI Controllers

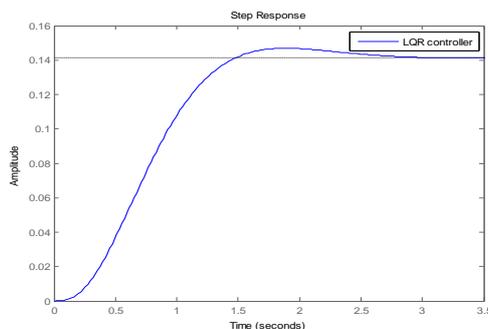


Figure 11: Performance of LQR Controller

The simulation results evaluated for the entire controllers based on the transient response specifications are depicted in Table 2.

Table 2: Simulation results

PARAMETERS	P	PD	PI	MPC	LQR
Rise Time (sec)	1.7142	1.7058	22.2959	0.0046	0.8879
Settling Time (sec)	5.7461	5.7102	77.3096	0.9998	2.3828
Overshoot (%)	11.7865	11.7804	11.4882	0.0000	3.8217

### 5. Conclusion

This paper has presented the review of the different performances of control algorithms on position control of DCSM. Among the proposed control strategies, simulation results indicate that MPC could be a better controller of position control of the DC servo motor. One of the difficulties encountered in tuning the MPC is getting the accurate prediction as the poor prediction would make the matter worst. Therefore, an Adaptive MPC

Based on some artificial intelligent e.g. Particle Swarm Optimization (PSO) or Bacterial Foraging Optimization Algorithm (BFOA) should be tested on the system to improve the settling time.

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