

Based on Wavelet Transform and Fractal Theory Image Compression Coding

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Abstract: This paper study the application of wavelet transform and fractal theory in image coding. The image coding based on wavelet transform has been studied firstly, Then, the paper make a research of fractal theory counterpart. After that, this paper research the hybrid compression coding based on wavelet transform and fractal theory, also make simulation experiment of three image coding ways mentioned above in the end, in which shows that the hybrid compression coding based on wavelet transform and fractal theory can effectively improve the image compression ratio because it make the best of the advantage of wavelet transform and fractal theory.

Keywords: Wavelet transform, Fractal theory, EZW coding, SPIHT coding.

The storage and transmission of data compression information is an important part of information technology. No matter what kind of transmission medium for transmission, will encounter a lot of image data transmission problems [1].For example, a pixel is 512 * 512Size of 0.46M images to 8k/s speed transmission to be nearly 60s time. In order to improve the speed of transmission of image data, we must look for more efficient image compression coding algorithm.

Since the late 1980s, wavelet transform has developed. This is a new signal processing methods, widely used in various fields. The concept of multi-resolution analysis of wavelet transform is proposed, Similar to binary carry by S. Mallat, S. Mallat has proposed a tower fast wavelet transform algorithm for signal decomposition and reconstruction[2].The wavelet transform has good local correspondence in frequency domain and time domain, You can define an image on any order of magnitude. Wavelet transforms by preserving the low frequency portion of the image, Remove the high frequency part, According to people's needs can achieve a higher compression ratio. Wavelet transform applications include signal processing image compression, computer vision and other fields.

The current wavelet transform theory has become more mature, Which has a huge application in the field of image compression. Appeared with the emergence of fractal theory, Fractal image compression coding is currently the most promising image compression coding, People around to shorten the coding time, Improve the image compression ratio and so on to make continuous improvement[3].It is also meaningful to study the role of wavelet transform and fractal theory in image compression coding.

2. Based on Wavelet Transform Image Compression Coding

2.1 Wavelet transform theory

(1)Continuous wavelet transform

The basic idea of continuous wavelet transform is to use a set of wavelet basis functions to represent a function or signal,It Will be any continuous function $f(t) \in L^2(R)$, Under the wavelet base to start, called this expansion as a function $f(t)$ Continue Wavelet Transform[4].

$$WT(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \Psi\left(\frac{t-b}{a}\right) dt \quad (1)$$

among them, For the expansion factor a , For the translation factor b , $\Psi(\)$ is Wavelet basis function.

(2)Discrete Wavelet Transform

Discrete wavelet transform is the discretization of continuous wavelet transform. Considering that the computer code is binary, Generally taken $a = 2^j$, For octave j , $b = n * 2^j$, n is a positive integer, The formula of discrete wavelet transform is [1-2].

$$DWT(2^j, n * 2^j) = \frac{1}{\sqrt{2^j}} \sum_k f(k) \Psi\left(\frac{t}{2^j} - n\right) dt \quad (3)$$

2.2 Application of Wavelet Transform in Image Compression Coding

At present, the classical wavelet image coding is mainly EZW (Embedded wavelet zero-tree image coding) 、SPIHT (Layered wavelet tree set segmentation algorithm) etc[5].

(1)EZW coding

EZW encoding is done after the wavelet decomposition of the image, Sub-images with different resolutions will appear, The lowest frequency of the offspring is in the upper left corner, The highest frequency of the offspring is in the lower right corner. A parent node (i, j) .there are 4 child nodes, Respectively include the following nodes: $(2i, 2j), (2i, 2j + 1), (2i + 1, 2j), (2i + 1, 2j + 1)$

By constructing a small quadtree structure. There is a great correlation between the decomposition coefficients at all levels, And then take absolute values for all wavelet coefficients, For a given threshold T , Corresponds to all $|x| < T$, It is said that x smaller than T , usually, x is the coefficient of descendants is not important enough, Rarely important is called isolated zero, Set these isolated zeros to zero.

(2)SPIHT coding

SPIHT (Multi - series aggregate splitting algorithm) The algorithm uses the tree structure and EZW. Using a zero-tree structure similar. The difference between the two algorithms is that SPIHT. The algorithm is very different in the segmentation of the coefficient subset and the transmission of important information, It can achieve a large amplitude of the first transmission[6]. For each node entered (i, j) . There are four children, $(2i, 2j), (2i, 2j + 1), (2i + 1, 2j), (2i + 1, 2j + 1)$.For image N-level wavelet decomposition, In EZW, LL_n have three children HL_n, LH_n, HH_n , use SPIHT. There are no children LL_n in the wavelet structure, The efficiency of the algorithm is further improved.

In addition, EBCOT (An Embedded block coding algorithm for optimal cut-off points) , It is also widely used in image compression coding.

3. Based on Fractal Theory

3.1 Fractal theory Image Compression Coding

Fractal image compression theory usually consists of four parts: Affine transformation、 Compression theorem、 Iterative function system and collage principle.

(1)Affine transformation

Affine transformation can be understood as matrix coordinate transformation, For example, affine transformation $W : R^2 \rightarrow R^2$ two-dimensional to two-dimensional transformation, Shaped like $W(x) = Ax + t$, among them W, A, t as a matrix. Only if when $r_\sigma(A) < 1$, Transform W is compression. There is the following relationship, $r_\sigma(A) \leq \|A\| \leq s$, So sometimes when $\|A\| \geq 1$ or $s \geq 1$, transform W eventually compressed.

(2)Compression theory

Each convergence image in the function space has a fixed point, So that each point in the function space passes through the contraction of the contiguous image. The resulting dottedline converge to this fixed point[7].

(3)Iterative function system(IFS)

Each iterative function system can constitute a contraction mapping in the function space. It is concluded that each iteration function system determines an image, which can be represented by affine transformations[8].

(4)The Collage Theorem

Given an image I, You can select N contraction images, This image is N transforms to get N sets of images. Each elephant set is a small image, If the N small images collage up the image and the distance between the image I any small. A elephant set up (IFS) [9].

3.2 Application of Fractal Theory in Image Compression Coding

The traditional fractal image compression method includes the following three processes, the process is as follows:

(1)Image segmentation

Splits an image $N \times N$ of size into a square block of equal size I^2 Splits an image of size into a square block of equal size, The square block is counted as a range block $R_i, R_i \cap R_j = \phi, \cup R_i = I^2$ [10]. Divide the image into small pieces that are not intersecting, While also dividing the image into larger chunks, Also known as domain D_i , The edge is twice as large as the range block.

(2) Create a search domain

Search for a domain block that is similar to the current definition D_i , A domain block that is as large as defining a domain block, Move the domain blocks horizontally and vertically. Horizontal and vertical direction of the move to h , This makes it possible to determine the size of the search field S_n :

$$S_n = \left(\frac{N - 2a}{h} + 1 \right)^2 \quad (4)$$

(3) Find the best match block

Found by (2) search $D_i, w(D_i) \rightarrow R_i$. After affine transformation, the effect of the closest affine transformation parameters recorded, thus completing the fractal compression of the image coding.

4. Hybrid compression coding based on wavelet transform and fractal theory

Wavelet transform coding converts data into wavelet coefficients to store, By removing the high frequency part of the image, It Can compress the image, But did not use the similarity between the wavelet domain. It's compression ratio can up to higher. For block similarity to the image, The fractal image coding makes the compression relatively large by using the similarity between the image blocks. And fractal coding algorithm complexity is relatively large, fidelity is not very good. In this paper, we use the similarity of the wavelet coefficients between the wavelet transform fields to improve the compression ratio of the image coding, and to preserve some details of the image, such as the edge part.

The image compression algorithm based on fractal theory in wavelet domain is studied. This algorithm combines the advantages of wavelet transform and fractal theory, and improves the efficiency of image compression.

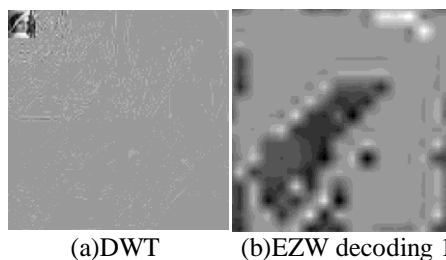
The algorithm is to transform the fractal compression coding of the time domain into the quantization coding of the wavelet domain. The algorithm is as follows [11].

- (1) The lowest resolution of the four sub-band images with no distortion or less distortion to be encoded.
- (2) The number of ranges and the number of domains are constructed, that is, the construct and (k is from the kth level).
- (3) For each Range tree to find the number of matches with it D_1^k . The matching parameter is the encoding result of the range tree. (It mainly includes some parameters of position, geometric transformation and rotation transformation, Thus forming a coding chain. Click the Range tree to finish.
- (4) The lowest resolution sub and image of the lowest resolution is recovered at the time of decoding, and then the sub and image of the resolution is presented by the range tree.

5. Experimental results

5.1 the Experiment of Based on Wavelet Transform Image Compression Coding

In this paper, EZW coding simulation experiments, a pixel for the $512 * 512$ of lena.bmp image encoding and decoding, through matlab programming simulation as shown in **Figure 1**.



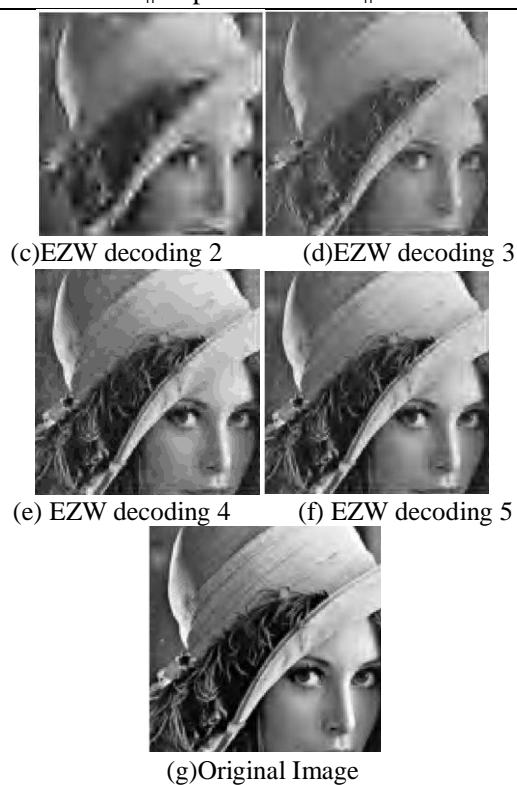


Figure 1: lena recovery

Figure 1 (a) is the lena.bmp according to DWT transform get the image. **Figure 1** (b) to **Figure 1** (f) is the EZW decoding image. The experimental results show that the improved wavelet transform coding (EZW coding) is faster than the direct decoding of the wavelet transform, and the program runs more faster.

5.2 Based on fractal theory image compression coding experiment

Split code on lena.bmp of image with 512 * 512 pixel is decoded and the result as shown in **Figure 2**.

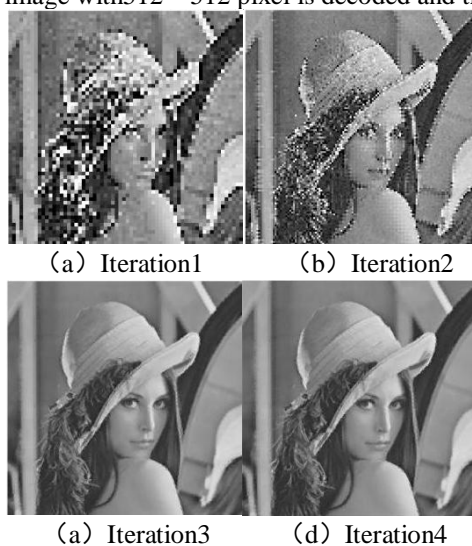


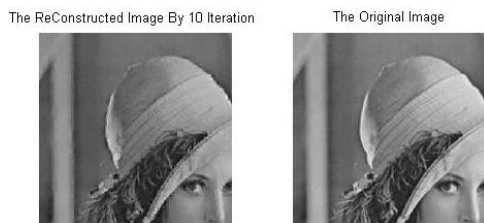


Figure 2: Iteration process

The results show that the lean.bmp image is iterated to obtain the **Figure 2** (a) to (h). Compared with the original image, the image once iteration, the more clear. Under normal circumstances Diego 8-15 times and the original map can be basically similar, but the program runs to spend 10 minutes, coding efficiency is not high.

5.3 Hybrid compressed coding experiment based on wavelet transform and fractal theory

In this paper, we study the lena.bmp image with a 512 * 512 pixel for fractal image coding and the pixels are larger than the 512 * 512 Pentagon and the Sun Yat-sen University campus. The results are shown in **Figure 3-Figure 5**.



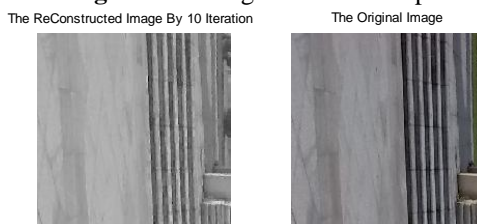
(a)

Figure 3: lean image iteration graph



(b)

Figure 4: Pentagon iteration map



(c)

Figure 5:School gate

Figure 3-Figure 5 give lean.bmp, the Pentagon and the final university of the final iteration map, The simulation results show that the image becomes close to the original image after 10 iterations, and the program runs for more than one minute. The program runs faster than the traditional fractal image compression coding. The validity of the algorithm is further verified.

In this paper, EZW coding, traditional fractal image compression coding and hybrid coding based on wavelet transform and fractal theory are used to encoding and decoding in the same image. It is much better to reflect whether the image compression coding has a high efficiency. The PSNR and encoding time are compared with those of the program. The specific results are shown in **Table 1**.

Table 1: experiment compare

	Imagecoding method	PSNR(db)	Coding time(s)
Lean.bmp	EZW	28.5	180.2045
	Traditional fractal image coding	34.2	601.3451
	hybrid Coding Based onWavelet Transform and Fractal Theory	35.1	69.6789

It can be seen from **Table 1** that the effect of hybrid image coding based on wavelet transform and fractal theory is best. the shortest time, PSNR highest. In the traditional fractal image coding, EZW coding is the worst, and the experimental results show that the application of fractal theory in image compression coding is significant.

6. In Conclusion

In this paper, We studied the based on wavelet transform image compression coding , based on fractal theory image compression coding and hybrid compression coding based on wavelet transform and fractal theory. The experimental results show that the hybrid compression coding based on wavelet transform and fractal theory can make full use of the advantages of wavelet transform and fractal theory, and effectively improve the image compression ratio. In the future, we will further study the combination of fractal theory and discrete cosine transform coding, the combination of neural network and fractal theory, and study the new metric similarity criterion, and reduce the distortion rate of reconstructed image under the premise of keeping the compression ratio.

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