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Radial Transmission Line Voltage Stability Analysis

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Abstract: QV and PV curves are most widely used voltage stability analysis tools today. The PV curve is formed by varying system load or transfer and plotting it against voltage. Analytical expression for complex power, real power has been formulated using exact representation of transmission line with ABCD parameters using elementary mathematics and receiving end circle diagram. The expression has been used for plotting PV curves at different power factor using MATLAB application software.
Keywords: Power Transfer, PV Curve, voltage stability.

1. Introduction

Transmission systems are more becoming more stressed due to increased loads and inter-utility power transfers. With growing size, along with economic and environmental pressures, the efficient operation of the power system is becoming increasingly threatened due to problem of voltage instability and collapse. QV and PV curves are most widely used voltage stability analysis tools today. The PV curve is formed by varying system load or transfer and plotting it against voltage. The PV curve can provide real power and voltage margins using the knee of the curves as reference point. PV curves at constant power factor are used to get maximum power transfer at critical voltage. The application of PV curve in voltage stability studies is enormous. Many proximity indicators are identified using PV curves. Voltage and power are controllable in upper region of the PV curve known as stable region.

2. Theoretical Background

A radial transmission line as shown in figure 1 in which a generator with constant voltage $V_S\angle\delta$ supplying complex power S_R to a load with terminal voltage $E\angle 0$ through a transmission line represented by its ABCD parameters.

Let the generalised line constants be expressed as $A\angle\alpha,\ B\angle\beta,\ C\angle c,\ D\angle\Delta$. The complex power at receiving end is given by the expression.

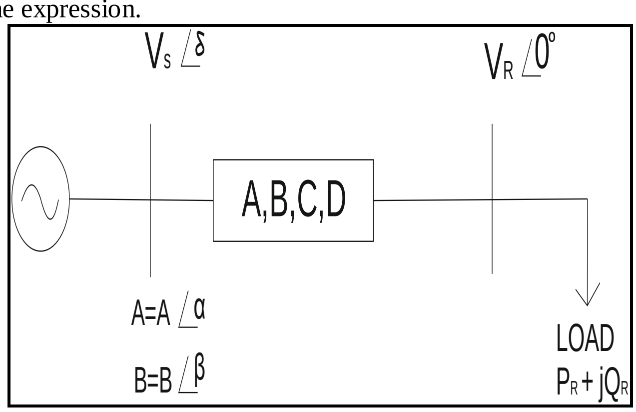

Fig.1: Radial Transmission Line

$$S_R = P_R + j\,Q_R = V_R\, I_R^* \quad (1)$$

Where I_R^* is the conjugate of receiving end current I_R. From two port representation of a transmission line we have

$$V_S = AV_R + BI_R \quad (2)$$

$$I_S = CV_R + DI_R \quad (3)$$

Thus receiving end current can be expressed as

$I_R = \frac{V_S - A\,V_R}{B} = \frac{V_S \angle\delta\ - A\angle\alpha\ V_R \angle 0°}{B\angle\beta} = \left(\frac{V_S}{B}\angle\delta - \beta\right) - \left(\frac{A}{B} V_R \angle\alpha - \beta\right)$ (4)

So conjugate of I_R,

$I_R^* = \left(\frac{V_S}{B}\angle\beta - \delta\right) - \left(\frac{A}{B} V_R \angle\beta - \alpha\right)$ (5)

Substitute the value of I_R^* in equation (1), we have

$S_R = P_R + j\,Q_R$

$S_R = \left(\frac{V_R V_S}{B}\angle\beta - \delta\right) - \left(\frac{A}{B} V_R^2 \angle\beta - \alpha\right)$ (6)

Separating real and reactive component we have

Receiving end real power $P_R = \left(\frac{V_R V_S}{B}\cos\beta - \delta\right) - \left(\frac{A}{B} V_R^2\ \cos\beta - \alpha\right)$ (7)

And receiving end reactive power,

$Q_R = \left(\frac{V_R V_S}{B}\sin\beta - \delta\right) - \left(\frac{A}{B} V_R^2\ \sin\beta - \alpha\right)$ (8)

The transmission lines are usually operated with constant sending end and receiving end voltages, so one component each of receiving and sending end powers is a fixed phasor while other component is a phasor of constant magnitude and variable angle. The loci of S_R and $S_S$are therefore, circles drawn from the tip of constant phasors or radius.

The circle drawn with receiving end real and reactive power components as the horizontal and vertical coordinates is called receiving end power circle diagram. The above equations (7) and (8) can be rewritten as:

$P_R + \left(\frac{A}{B} V_R^2\ \cos\beta - \alpha\right) = \left(\frac{V_R V_S}{B}\cos\beta - \delta\right)$ (9)

$Q_R + \left(\frac{A}{B} V_R^2\ \sin\beta - \alpha\right) = \left(\frac{V_R V_S}{B}\sin\beta - \delta\right)$ (10)

Squaring and adding equations (9) and (10) we have

Or, $[\,P_R + \left(\frac{A}{B} V_R^2\ \cos\beta - \alpha\right)]^2 + \left[\,Q_R + \left(\frac{A}{B} V_R^2\ \sin\beta - \alpha\right)\right]^2 = \left[\frac{V_R V_S}{B}\right]^2$

Above equation represent equation of circle $[\,(x-h)^2 + (y-k)^2] = r^2$ (11)

Where, x-coordinate of the centre of circle (h) $= -\left(\frac{A}{B} V_R^2\ \cos\beta - \alpha\right)$ and y-coordinate of the centre of circle (k) $= -\left(\frac{A}{B} V_R^2\ \sin\beta - \alpha\right)$ and radius of the circle $= \frac{V_R V_S}{B}$, as we know complex power at receiving end is given by

$S_R = P_R + j\,Q_R = \left(\frac{V_R V_S}{B}\angle\beta - \delta\right) - \left(\frac{A}{B} V_R^2 \angle\beta - \alpha\right)$ (12)

The above equation represent a circle for varying value of δ with position of centre indicated by $-\left(\frac{A}{B} V_R^2 \angle\beta - \alpha\right)$ and radius by $\frac{V_R V_S}{B}$, where A, B, C, D are generalised line constant and δ is power angle.

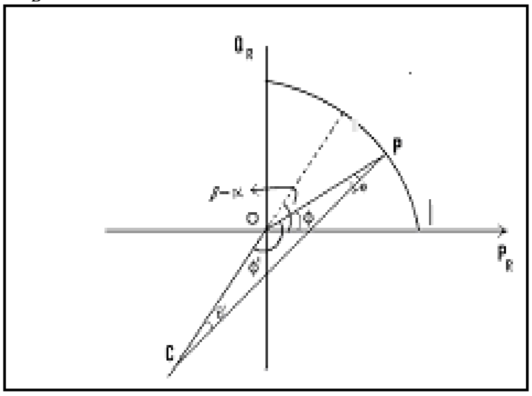

Fig.2: Receiving end circle diagram

From figure 2

$$\mathrm{OC} = \frac{A}{B} V_R^2 \quad (13)$$

$$\mathrm{OP} = S_R \quad (14)$$

$$\mathrm{CP} = \frac{V_R \mathrm{V_S}}{\mathrm{B}} \quad (15)$$

$$\delta' = \delta - \alpha \quad (16)$$

$$\phi' = 180° - (\beta - \alpha) + \phi \quad (17)$$

Φ is the power factor angle and is positive for lagging power factor and negative for leading power factor.

In ΔOCP

$$\frac{OP}{\sin \delta'} = \frac{CP}{\sin \phi'} = \frac{OC}{\sin \theta} \quad (18)$$

From above equation (12) to (18)

$$S_R = \frac{V_S^2 \sin \theta \sin \delta'}{AB \sin^2 \phi'} \quad (19)$$

Also from ΔOCP

$$\theta = 180° - (\phi' + \delta') \quad (20)$$

Therefore,

$$S_R = \frac{V_S^2 \sin(\phi' + \delta') \sin \delta'}{A\, B \sin^2 \phi'} \quad (21)$$

For S_R to be maximum

$$\frac{dS_R}{d\delta} = \frac{dS_R}{d\delta'} = 0$$

Solution of (21) provides critical value of power angle δ, critical value of voltage and maximum value of complex power.

$$\delta'_{cr} = 90° - \frac{\phi'}{2}, \text{ and } \delta_{cr} = 90° - \frac{\phi}{2} + \alpha \quad (22)$$

$$S_{R\,max} = \frac{V_S^2}{4\, A\, B \sin^2 \frac{\phi'}{2}} \quad (23)$$

$$V_{R\,cr} = \frac{V_S}{2A \sin \frac{\phi'}{2}} \quad (24)$$

Equation (21) and (22) relate complex power with maximum complex power

$$S_R = \frac{S_{R\,\max} [\sin(\phi' + \delta') \sin \delta']}{\cos^2 \phi'/2} \quad (25)$$

The maximum value of active power and limiting value of reactive power $Q_{R\,lim}$

$$P_{R\,max} = S_{R\,\max} \cos \phi \quad (26)$$

$$Q_{R\,lim} = S_{R\,\max} \sin \phi \quad (27)$$

Receiving end voltage is obtained as

$$V_R = \frac{V_S \sin(\phi' + \delta')}{A \sin \phi'} \quad (28)$$

3. Case Study

To study the loadability of a radial transmission line, consider a 345 KV three phase line with following parameters. $A = .950\angle.33°$ and $B = 99.96\angle 83.8°$, sending end voltage is constant $V_S = 345\,KV$. Table I gives the values of ϕ, ϕ', $S_{R\,max}$, $P_{R\,max}$, $V_{R\,cr}$, and $\delta_{R\,cr}$. Limiting value of reactive power is also listed in Table I.

Table I

Maximum PV Table

With given data and using above equations we can determine the value of S_R, V_R, P_R that are given in following table for different power factors.

ϕ Power Factor Angle	ϕ' Degree	Srmax MVA	Prmax MW	Qrlim MVAr	Del critical degree	Vr critical kv
-20	76.53	825.26	775.49	-282.25	51.73	293.20
-10	86.53	673.81	663.57	-117.00	46.73	264.93
0	96.53	568.39	568.39	0	41.73	243.32
10	106.53	492.81	485.32	85.57	36.73	226.57
20	116.53	437.57	411.18	149.66	31.73	213.49

Table II

At (- 20 Degree) Leading Power Factor Angle PV Table

del_1 Degree	Sr MVA	Pr MW	Vr KV
56.73	815.09	765.93	271.92
46.73	815.09	765.93	312.24
61.73	784.89	737.56	248.58
41.73	784.89	737.56	328.90
66.735	735.58	691.22	223.35
36.735	735.58	691.22	343.06
71.735	668.66	628.34	196.42
31.735	668.66	628.34	354.61

Table III

At (-10 Degree) Leading Power Factor Angle PV Table

del_1 Degree	Sr MVA	Pr MW	Vr KV
51.73	664.16	654.07	242.19
41.73	664.16	654.07	285.65
56.73	635.49	625.84	217.60

36.73	635.49	625.84	304.20
61.73	588.69	579.74	191.36
31.73	588.69	579.74	320.44
66.73	525.16	517.19	163.67
26.73	525.16	517.19	334.24

Table IV
At (0 Degree) Unity Power Factor Angle PV Table

del_1 $Degree$	Sr MVA	Pr MW	Vr KV
46.73	558.64	558.64	218.62
36.73	558.64	558.64	266.17
51.73	529.71	529.71	192.26
31.73	529.71	529.71	286.99
56.73	482.47	482.47	164.43
26.73	482.47	482.47	305.63
61.73	418.34	418.34	135.36
21.73	418.34	418.34	321.94

Table V
At (10 Degree) Lagging Power Factor Angle PV Table

del_1 $Degree$	Sr MVA	Pr MW	Vr KV
41.73	482.35	475.02	199.25
31.73	482.35	475.02	252.17
46.73	451.27	444.42	170.41
26.73	451.27	444.42	275.84
51.73	400.53	394.45	140.28
21.73	400.53	394.45	297.42
56.73	331.67	326.63	109.07
16.73	331.67	326.63	316.74

Table VI

At (20 Degree) Lagging Power Factor Angle PV Table

del_1 Degree	Sr MVA	Pr MW	Vr KV
36.73	425.56	399.90	182.59
26.73	425.56	399.90	242.77
41.73	389.88	366.37	150.31
21.73	389.88	366.37	270.20
46.73	331.63	311.63	116.87
16.73	331.63	311.63	295.57
51.73	252.56	237.33	82.55
11.73	252.56	237.33	318.69

The PV curve for the above problem has been drawn for $20°$ lagging, $10°$ lagging, $0°$ and $10°$ leading, $20°$ leading power factor angle. The algorithm used is given below

(i) Compute del_1 ($\delta^{'}$) = $\delta_{cr} \pm \Delta\delta$, δ_{cr} is obtained from TABLE I, $\Delta\delta$ is change in value of δ from $5°$ to $20°$.
(ii) Compute S_R
(iii) Compute P_R
(iv) Compute V_R

Figure 3 shows PV curve at different power factor angle, where negative represent leading angle.

4. Result and Conclusion

From the curve and table we observed that real power (loadability) increases from lagging to leading power factor. We also obtain two values of receiving end voltage V_R for given P_R except at $P_{R\,max}$ .The knee point is shifted towards higher real power and higher voltage. This shows that the voltage stability improves as the power factor moves from lagging to leading loads as shown in figure 3. Figure 4 shows variation of maximum receiving end apparent power, receiving end maximum real power, receiving end limiting reactive power and critical receiving end voltage with respect to power factor angle (degree). Negative value represent leading power factor

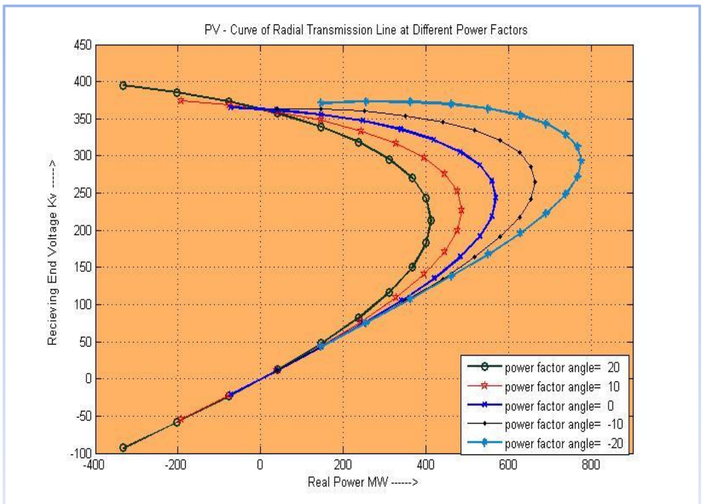

Fig.3: shows PV curve at different power factor angle

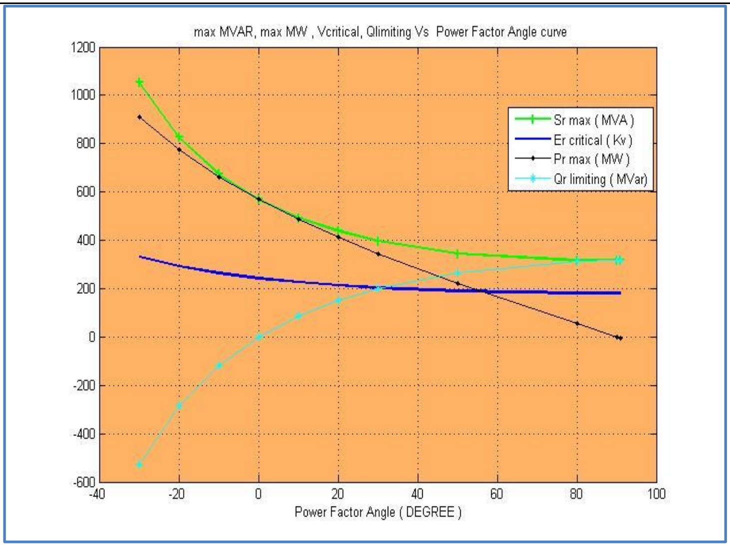

Fig.4: variation of Sr_max, Pr_max, Qr_lim, Vr_critical
From leading to lagging condition

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