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A Method for Piecewise-point Derivative of Piecewise Function

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Abstract: Usually, we discuss the piecewise derivative using the definition of derivative. However, it increase the computational difficulty and computation efficiency. In this paper, a method is proposed for the derivatives of piecewise function which the neighborhood limitation exists using the derivatives formulas. Furthermore, it points out that the limitation existence is only a sufficient condition for the derivatives at the piecewise point. Finally, we use the test problems to measure and evaluate the proposed method and the results indicate that the proposed algorithm can effectively solve the problem.

key word: Piecewise function； Piecewise-point； Derivative

1. Introduction

In the process of computing derivative, the derivative of the piecewise function at the piecewise point is difficult. Usually, we solve the derivative of piecewise function at the piecewise point using derivative definition or the definition of right and left derivatives[1]. However, the process of the first method is tedious and the computational burden of the second method is great. In this paper, a method is proposed for the derivative of piecewise function which the neighborhood limitation exists and the results indicate that the proposed algorithm can effectively solve the problem.

2. The left and right neighborhood derivative limits are existence at the piecewise point

If the piecewise function is continuous at the segment point x_0, the left neighborhood $(x_0-\delta, x_0)$ and the right neighborhood $(x_0, x_0+\delta)$ are differentiable which can be solved by theorem 1 and theorem 2.

Theorem 1 If the piecewise function $f(x)$ is differentiable in the neighborhood $\overset{o}{U}(x_0)$ at the piecewise point x_0, satisfy:

(i) The piecewise function $f(x)$ is continuous at the piecewise point x_0;

(ii) $\lim\limits_{x\to x_0^-} f'(x)$ and $\lim\limits_{x\to x_0^+} f'(x)$ are existence and have the same value A.

Then, $f'(x_0)$ exists and $f'(x_0)=A$.

Proof. For $f(x)$ is differentiable at every point in the neighborhood $\overset{o}{U}(x_0)$, and which is continuous at x_0, then,

$$\frac{f(x)-f(x_0)}{x-x_0}=f'(\xi_1) \qquad (x\in(x_0-\delta,x_0),\xi_1\in(x,x_0)).$$

$$\frac{f(x)-f(x_0)}{x-x_0}=f'(\xi_2) \qquad (x\in(x_0,x_0+\delta),\xi_2\in(x,x_0)).$$

Because $\lim\limits_{x\to x_0^-} f'(x)=\lim\limits_{x\to x_0^+} f'(x)$=A, so

$$f'_-(x_0)=\lim_{x\to x_0^-}\frac{f(x)-f(x_0)}{x-x_0}=\lim_{x\to x_0^-} f'(\xi_1)=\lim_{\xi_1\to x_0^-} f'(\xi_1)=A,$$

$$f'_+(x_0)=\lim_{x\to x_0^+}\frac{f(x)-f(x_0)}{x-x_0}=\lim_{x\to x_0^+} f'(\xi_1)=\lim_{\xi_2\to x_0^+} f'(\xi_2)=A,$$

so $f'(x_0)=f'_-(x_0)=f'_+(x_0)=A$.

Example 1[2] Discuss the derivative of $f(x)=\begin{cases}\sin x & x<0\\ x & x\geq 0\end{cases}$ at point $x=0$.

Solution: Because $f(x)$ is continuous at point $x=0$, and there are

$$\lim_{x\to 0^-} f'(x)=\lim_{x\to 0^-}\cos x=1 \qquad \lim_{x\to 0^+} f'(x)=\lim_{x\to 0^+}1=1$$

then, $f'(0)$ exist and $f'(0)=1$.

In theorem 1, it is worth noting that the existence of $\lim\limits_{x\to x_0^-} f'(x)$ and $\lim\limits_{x\to x_0^+} f'(x)$ can not guarantee the existence of the derivatives at the piecewise point.

Example 2[3] Discuss the derivative of function $f(x)=\begin{cases}x+1 & x<0\\ x & x\geq 0\end{cases}$ at point $x=0$.

Solution: Because $\lim\limits_{x\to 0^-} f(x)\neq\lim\limits_{x\to 0^+} f(x)$, so $f(x)$ is not continuous at point $x=0$ and are not be differentiable, but

$$\lim_{x\to 0^-} f'(x)=\lim_{x\to 0^+} f'(x)=1$$

So, theorem 1 is used when it satisfied the two conditions at the same time. According to Theorem 1, we can obtain the corollary 1 when the left and right limitations are not equal.

Corollary 1 The piecewise function $f(x)$ is guided within a $\overset{\circ}{U}(x_0)$ of the picewise-point x_0, if both $\lim\limits_{x\to x_0^-} f'(x)$ and $\lim\limits_{x\to x_0^+} f'(x)$ are present but not equal, then $f(x)$ is not directed at point x_0.

Example 3[4] Discuss the derivatives of function $f(x)=\begin{cases}x^2 & x\geq 0\\ -x & x<0\end{cases}$ at point $x=0$.

Solution: Because $\lim\limits_{x\to 0^-} f'(x) = \lim\limits_{x\to 0^-} -1 = -1$, $\lim\limits_{x\to 0^+} f'(x) = \lim\limits_{x\to 0^+} 2x = 0$,

$$\lim_{x\to 0^-} f'(x) \neq \lim_{x\to 0^+} f'(x)$$

so $f(x)$ can not be made at point $x = 0$.

3. At least one of the left and right neighborhood derivative limits at the piecewise point

If the piecewise function $f(x)$ is continuous at the piecewise point x_0 and the derivative of $(x_0 - \delta, x_0)$ and $(x_0, x_0 + \delta)$ are not existence at the point x_0, we can not determine the differentiability of the point x_0. Sometimes, we should solve the derivative of the point x_0 by using the definition of the derivatives.

Example 4[5] Discuss the derivative of function $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at point $x = 0$.

Solution: $f'(0) = \lim\limits_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x\to 0} \dfrac{x^2 \sin\frac{1}{x} - 0}{x - 0} = \lim\limits_{x\to 0} x \sin\frac{1}{x} = 0$ .but $\lim\limits_{x\to 0} f'(x) = \lim\limits_{x\to 0}(2x\sin\frac{1}{x} - \cos\frac{1}{x})$ does not exist.

From Example 4, we can know that the piecewise point may also exist when the left and right neighborhood derivatives limitations do not exist at the piecewise point. It shows that the derivatives limitation at the piecewise point within the left and right neighborhood is only sufficient conditions for the existence of the derivatives at the piecewise point.

4. Conclusion

In this paper, we can judge the derivatives of the segment function at the piecewise point based on theorem 1 and corollary 1. It overcomes the calculation difficulty and greatly improves the efficiency of the calculation compare with solving this problem using the definition of derivatives. In additional, we also point out the sufficient condition that the left and right neighborhood derivatives of the piecewise point are only derivable.

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