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Application of Markov Chain Analysis Model for Predicting Monthly Market Share of Restaurants

Amare Matebu Kassa[1], Estar Abrham[2], Tsehay Seid[3]

[1]*Bahirdar University, Faculty of Mechanical and Industrial Engineering, Bahirdar, Polytechnic Institute, Ethiopia*

[2]*Faculty of Mechanical and Industrial Engineering, Bahirdar University, Amhara Region, Bahirdar, Ethiopia*

[3] *Faculty of Mechanical and Industrial Engineering, Bahirdar University, Amhara Region, Bahirdar, Ethiopia*

Abstract: The main objective of this study is to determine the market share of the restaurants around a specific area named Bahirdar technology institute of the Bahirdar University using Markov chain analysis model. A survey on four restaurants has been done using survey questionnaire and semi-structured interviews. The computation results of the market share for student restaurant, Poly zone restaurant, Fikir restaurant and other restaurants at the end of month 3 are 8.45%, 37%, 26.5 % and 28.32% respectively. The mathematical model portrays that by month 3, the market share of student restaurant decreased from 20% to 8.45%, the market share of Poly zone restaurant decreased from 45% to 37%, the market share of Fikir restaurant slightly decreased from 30% to 26.5%% but that of other restaurants increases from 5% to 11.40%. At equilibrium or steady state condition, the market share of the four restaurants in Poly campus at the long-run are 0%, 0%, 0%, and 100% respectively. Clearly, the three restaurants are dominated by the fourth restaurants named as other restaurants. Furthermore, the Markov chain analysis model could forecast the market share and profitability of the restaurants for the next consequent months.

Keywords: Markov chain analysis, market share, restaurants

1. Introduction

Markov chains have been widely studied and applied in brand switching studies and market share forecasting [1, 2]. The Markov brand switching model studies customer loyalty and forecasts the brands, products, or service that a customer is likely to purchase next. As aggregation of individual customers, market shares of companies and their competitors can also be studied using similar approach. Most applications used first order Markov chain to model the probability that a customer switching from one provider to another over a period of time, and to forecast future market shares. Previous studies has indicated that the Markov chain model was used to investigate the movements of a group of buyers among a number of sellers, and market shares as proportions are used in the state vectors.

In the theory, Markov chains is considered as the result of any transitions depend on the transition result, which precedes directly and only from it, transition which can be either a win or a loss. All crossing probabilities are grouped in the form of a transition matrix or crossing probabilities matrix, with the number of lines and columns equal to the number of possible states. It represents the connection between two consecutive phases, made according to the fidelity coefficient and reorientations.

Markov Chain model is widely applicable to forecast market demand of the customers in service industries such as Banking, Hotels/restaurants, telecommunications, etc. To solve problems using Markov chain models, it is necessary to build an initial distribution vector consists of initial possibilities of the system, having a static character specific to a particular moment in its evolution. Determining the products evolution on the market using Markov chains, require only knowledge of present state and transition probabilities. Two states of Markov process are known: recurrent state and transient state. The recurrent state is determined by the certitude that a process will return at a certain known state in a future moment. If there is a possibility that the process was no longer in that phase, then the state is known as transient. Markov chains are often used to model probabilistic systems when the chance of a future event is only dependent on the present and not dependent on the past. Markov chain method is a very good alternative, provided by economic-mathematical modeling, to study evolutions in the market products in relation with those of competitors. The advantage of this method is easily getting of some relevant information, which may contribute, to the improvement of marketing and strategic management to facilitate the activity through the development of strategies based on the product life cycle.

Predicting future states involves knowing system's likelihood or probability of changing from one state to another. These probabilities can be collected and placed in a matrix. Such matrix is also called the matrix of transition probabilities and shows the likelihood that the system will change from one state time period to the next. This enables to predict future states (conditions). As mentioned above the movement of a system from one state to another depends upon the immediately preceding state with constant probability forms the basis of Markov chain. However, before formulating a Markov chain model, following properties must be satisfied:

- There is finite number of possible states.
- States are both collectively exhaustive and mutually exclusive.
- The transition probabilities depend only on the current state of the system, i.e. if current state is known, the conditional probability of the next state is independent of the states prior to the present state.
- The long-run probability of being in a particular state will be constant over time.
- The transition probabilities of moving to alternative states in the next time period, given a state in the current time period must sum to 1.

Using Markov model one can study the evolution of variables in the future by analyzing their present behavior in the form of processes that take place in succession. In other words present determine the future. This model is based on a system, which in each phase is in a state, which belongs of a lot states. According to the model, each result is associated with a probability (p), which determines the transition from one state to another, transition from one current state (**0**) to another (1) is based on transition probabilities noted with pij. If i = j it presents fidelity for initial state, and in the situation in which the system is located in the same state during a period then it remained in its original state = i. Crossing probabilities influence the long term behavior of the system as well as his behavior in the short term.

In this study, Markov chain model is applied to estimate the customers' preference of restaurants in a specific area around Bahirdar technology institute. The study is carried out to determine the market share of these restaurants. It would be useful for the restaurants to know what qualities or variables attract the customers most; as a way of finding out optimum marketing mix that will lead them to profit optimization and the effect of customers' retention and loss on key business variables such as market share and profitability of the four restaurants.

2. Methodology

The Markov Chain is a stochastic process as a mathematical model for a system or an element that has random outcomes. These outcomes are viewed as a function of independent variables such as a temporal or spatial factor.

Around Bahirdar technology institute, there are more than ten restaurants that deliver a restaurant services for the customers. For this study, four categories of restaurants have been considered to examine their customers' loyalty, retention and loss. These are student restaurant (SL), poly zone restaurant (PR), Fikir restaurant (FR) and other restaurants (AR). The fourth category includes all the remaining restaurants. Sufficient data have been collected from the customers and service providers of those four categories of restaurants around Bahirdar technology institute of the Bahirdar University. A pilot survey questionnaire has been distributed to 120 customers of those restaurants adopting the method of simple random sampling technique over a period of two months (December, 2016 and January, 2016). Out of the 120 questionnaires distributed, 100 are completed and returned. The questionnaire administered for the research attempted to find out the decision of the customers' preference for choosing a particular restaurant among the four restaurants that serve customers and having the highest market share around Bahirdar technology institute campus. The average daily demand of the customers has been considered for the above mentioned months.

Markov chain model- has been used to predict customers' loyalty, customers' retention and loss and forecast the market share of the restaurants after n periods. POM software (Markov analysis module), as the number of periods increase, further changes in the state probabilities becomes smaller. So POM software has been used to see how these proportions would change after each day till they reach the steady state.

In order to analyze customers' loyalty and market share of the selected restaurants over a period of time using Markov Chain Method, the following procedures have been followed. First identify the possible outcomes (i = 1, 2 ... m) of each of the sequence of experiments or events (in this illustration events are services and the possible outcomes are the four restaurants). Second calculate P_{ij} that is the probability of being in state 'j' in the future given the current state i. The outcomes S_1, S_2,…,Sm are called states and the numbers P_{ij} are called transition probabilities. The transition matrix of the Markov chain, $P_{ij} = 0$ when no transition occurs from state i to state j; and $P_{ij} = 1$when the system is in state 'i', it can move only to state 'j' at the next transition. Third predicting future states involves knowing system's likelihood or probability of changing from one state to another. These probabilities will be collected and placed in a matrix called the matrix of transition probabilities (transition matrix) in constructing the transition matrix. Then, determine retention probabilities, gains and losses probabilities of each restaurant around Poly area, and develop matrixes of transition probabilities. Since there are four states (restaurants) selected around Bahirdar technology institute, the state space is in the form, S = {Student restaurant, Poly zone restaurant, Fikir restaurant and other restaurants}. As a result of the survey conducted on the customers including university students, the data collected about the relationship between their existing and next restaurant preferences are transformed into a Markov matrix.

3. Results and Discussion

A market survey in December, 2016 revealed that 45% of customers served at Poly zone restaurant, 30% of customers at Fikir restaurant, 20% of customers at Student restaurants, and 5% of customers at other restaurants. In January, from those who had been served in student's restaurant in the previous month, 15% go to poly zone restaurant, 20% go to Fikir restaurant, and 65% go to student's restaurant. From those who had been served in poly zone restaurant, 24.44% go to Fikir restaurant, 11.12% go to other restaurants and 64.44% go to poly zone. From those who had been served at Fikir restaurant, 30% go to poly zone, 23.33% go to other restaurant and 46.67% go to Fikir restaurant. From those who served at other restaurant, all of them go to other restaurant.

The current state of the students depends only on their immediately preceding state. At a given observation period, say nth period, the probability of the customer being in a particular state depends on its status at the n-1 period. The customers may be in one of the four possible states. These states describe all possible conditions of the customers. Every customer has four possible states and at any one time, every customer is in one and only one of its four states.

Let us describe:

State 1= student restaurant
State 2= Poly zone restaurant
State 3= Fikir restaurant
State 4= other restaurants

Events – choosing a restaurant to be served.

Transition probabilities:

P_{ij} = 0 when no transition occurs from state i to state j;

P_{ij} = 1 when the system is in state i, it can move only to state j at the next transition.

The matrix of transition probabilities are summarized on figure 1

January, 2016

December, 2016	INITIAL	SL	PR	FR	AR
SL.	20	13	3	4	0
PR.	45	0	29	11	5
FR.	30	0	9	14	7
AR.	5	0	0	0	5

Figure 1 The state transition matrix interms of retention and loss

In December 2016, on daily average 20 customers are served at student restaurant, 45 customers at poly zone restaurant, 30 customers at Fikir restaurant and 5 customers at other restaurants. In January 2016, among the 20 customers who had been served, Student restaurant has lost 3 customers to Poly zone restaurant, 4 customers to Fikir restaurant, none of them by other restaurants and 13 customers retain back to student restaurant. And among the 45 customers, who had been served in it, Poly zone restaurant has lost 11 customers by Fikir restaurant, 5 customers by other restaurants, none of them by student restaurant and 29 customers retain back to Poly zone restaurant. Among the 30 customers who had been served in it, Fikir restaurant has lost 9 customers by poly zone restaurant, 7 customers by another restaurant, none of them by student restaurant and 14 customers retain back to Fikir restaurant. From those who had been served at other restaurant, all of them retain back to other restaurants.

In January 2016, student restaurant has gained none of the customers (0%) from Poly zone restaurant, none of the customers (0%) from Fikir restaurant, none of the customers (0%) from other restaurants and 13 customers (65%) retain back from itself (student restaurant). And Poly zone restaurant has gained 9 customers (24.44%) from Fikir restaurant, 5 customers (11.12%) from other restaurants, none of them (0%) from students restaurant and 29 customers (64.44%) retain back from poly zone restaurant. Fikir restaurant has gained 11 customers (3%) from poly zone restaurant, 4 customers (0 %) from student restaurant, none of the customers (0%) from other restaurants, and 14 customers (46.67%) retain back to Fikir restaurant. From those who had been served at other restaurants, all of them (100%) retain back to other restaurants.

January, 2016

December, 2016

	INITIAL	SL	PR	FR	AR
SL.	20	13	0	0	0
PR.	45	3	29	9	0
FR.	30	4	11	14	0
AR.	5	0	5	7	5

Figure 2 The state transition matrix interms of retention and gain

In a matrix of transition probabilities, retentions are shown in as values on the main diagonal. The rows in the matrix show the retention and loss of customers while the columns show the retention and gain of customers.

January, 2016

December, 2016

	INITIAL	SL	PR	FR	AR
SL.	20	0.65	0.15	0.2	0
PR.	45	0	0.6444	0.2444	0.1112
FR.	30	0	0.3	0.4667	0.2333
AR.	5	0	0	0	1

Figure 3 Transition Probabilities Matrix of the four restaurants

The first column represents state of serving at student restaurant, the second column represents state of serving at poly zone restaurant, the third column represents state of serving at Fikir restaurant, and the fourth column represents state of serving at the other restaurant. Similarly the rows respectively represent serving at student restaurant, at poly zone restaurant, Fikir restaurant and at other restaurant. In general, there are four numbers of possible outcomes (state 1, state 2, state 3 and state 4) of the movement of customers from one state to another.

P_{ij} = probability of being in state j in the future given the current state i. The process can begin in one of the four possible states, and then possible to calculate probability of states relating to the overall sequence of events. Each step represents a time period (or condition) which results in another possible state.

From the given restaurants, the fourth one named as **other restaurants** denoted as $P_{44} = 1$ is an absorbing state**,** since it cannot leave that state after reaching it. The remaining states: student restaurant, Poly zone restaurant and Fikir restaurant are called transient states. On the other hand, *the above results can be represented by transition diagram.* The arrows from each state indicate the possible states to which a system can move from the given state. The transition diagram shows the four states and the probabilities of going from one state to another.

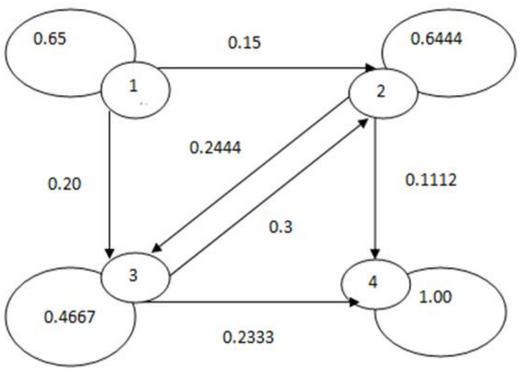

Figure 4 Transition diagram of the four restaurants

3.1 Probability tree diagram

The probability transition matrix can be illustrated using the probability tree diagram. Therefore, the transitions among the restaurants could be represented by the tree diagram. In the probability tree diagram, the circle represents the state at the beginning of a transition. These trees can also be used to evaluate and determine the probability that a student would be in any particular state at any particular time in the future, given the current state.

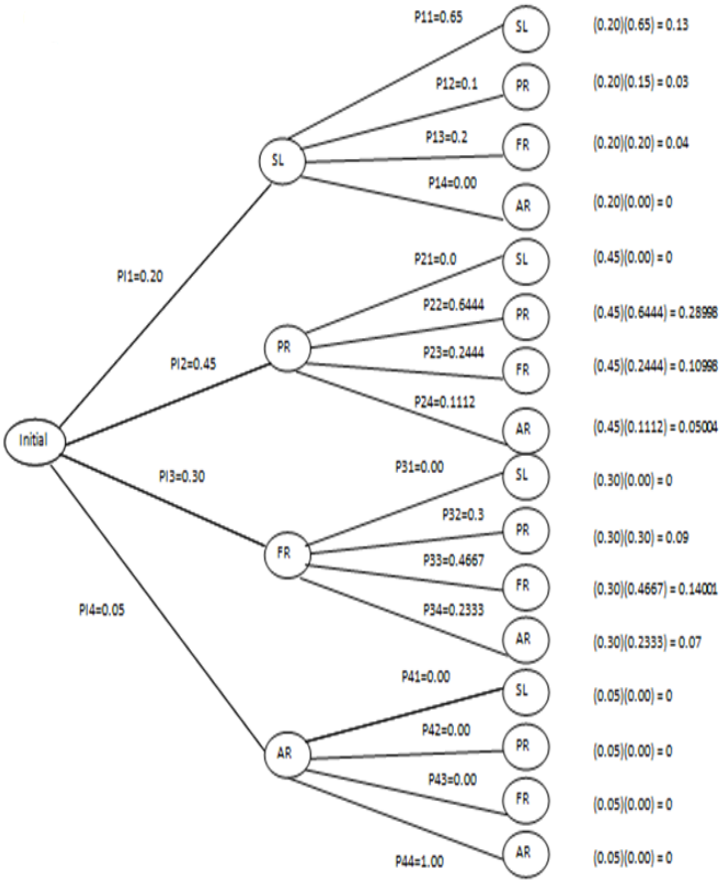

Figure 5 One month transition probability

The probability tree diagram in figure 5 represents two possible outcomes from an experiment with their assumed probabilities of occurrence from one step to another, along with branches that may connect them over a period of time.

The proportion of customers in state 1 after one day, add the numbers:

0.13 + 0.00 + 0.00 + 0.00 = 0.13

In a similar way, the proportion of customers in Poly zone after one day is

0.03 + 0.28998 + 0.09 + 0.00 = 0.40998

The proportion of customers in Fikir restaurant after one day is

0.04 + 0.10998 + 0.14001 + 0.00 = 0.28999

And the proportion of customers in another restaurant is

0 + 0.05004 + 0.07 + 0.00 = 0.12004

The initial distribution of states, 20%, 45%, 30% and 5% becomes 13%, 41%, 29% and 17%.

In a similar way, it is possible to calculate the probability of customers served in the four categories of the restaurants in month 2, month 3, month 4, and so on. For illustration purpose, the researcher has considered Poly zone restaurant to forecast the third month (February, 2016) market share as follows.

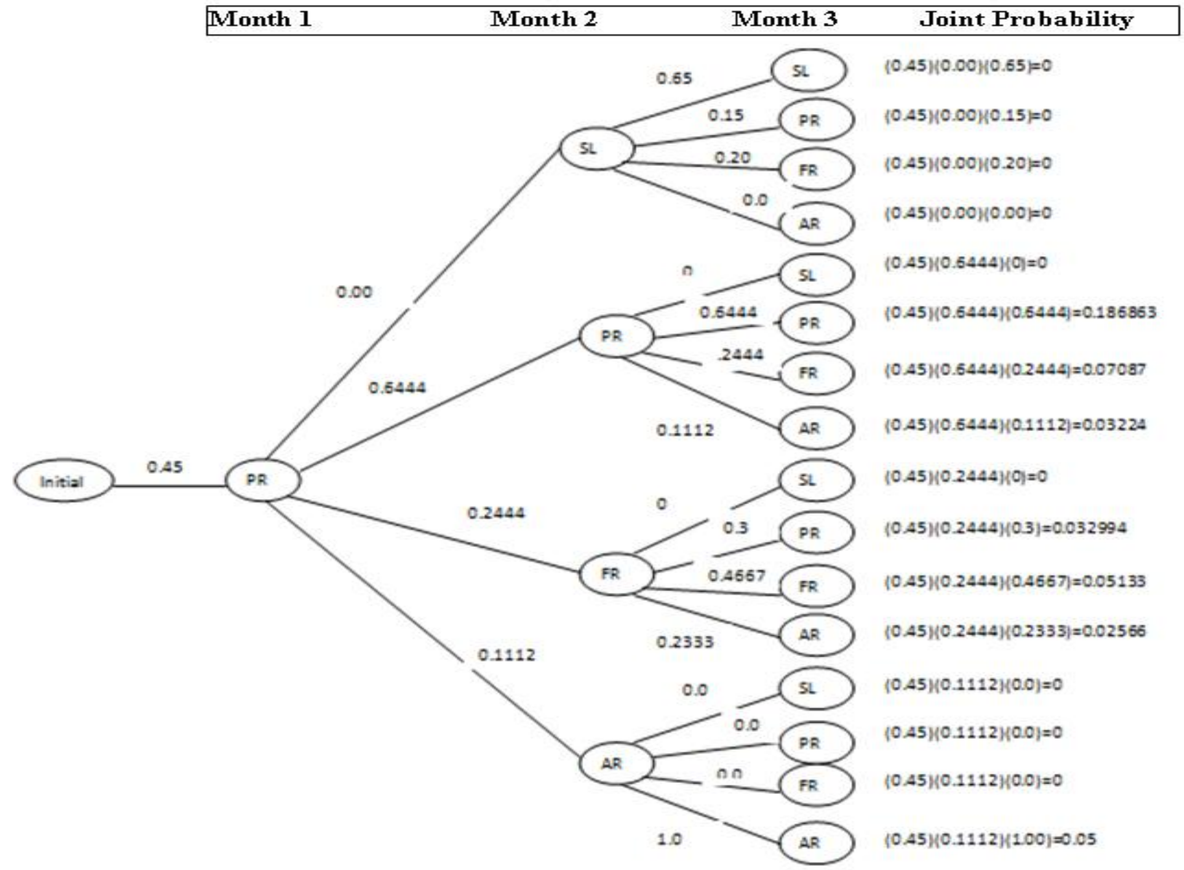

Figure 6 probability of customers served with Poly zone restaurant in month

The proportion of customers that are going to be served in student restaurant after two months transition or at the third month, from those who are served at Poly zone restaurant in month 1 (present) are computed as follows. Transition probabilities (From SL to SL, from PR to SL, from FR to SL, from AR to SL) = 0.0 + 0 + 0 + 0 = **0**

To compute the proportion of customers that are going to be served in Poly zone restaurant after two months transition or at the third month, from those who are served at poly zone restaurant in month 1(present) are computed as follows.

- Transition probabilities (From SL to PR, from PR to PR, from FR to PR, from AR to PR) = 0 + 0.186863 + 0.032994 + 0 = **0.22**
- The proportion of customers that are going to be served in Fikir restaurant after two months transition or at the third month, from those who are served at poly zone restaurant in month 1(present) are computed as follows. Transition probabilities (From SL to FR, from PR to FR, from FR to FR, from AR to FR) = 0 + 0.07087 + 0.05133 + 0 = **0.1222.**
- The proportion of customers that are going to be served in Fikir restaurant after two months transition or at the third month, from those who are served at Poly zone restaurant in month1(present) are computed as follows. Transition probabilities (From SL to FR, from PR to FR, from FR to FR, from AR to FR) = 0 + 0.03224 + 0.02566 + 0.05 = **0.1079**

3.2 Summary of the state probability distribution of customers in month 3

The probability distributions of customers for the four restaurants in the third month are computed and summarized as follows.

- The proportion of customers that are going to be served in student restaurant after two months transition or at the third month (from those who are served at student restaurant, poly zone restaurant, Fikir restaurant, and at other restaurant) in month 1 (present) is equal to 0.0845 + 0+ 0+ 0 = **0.0845.**
- The proportion of customers that are going to be served in Poly zone restaurant after two months transition or at the third month (from those who are served at student restaurant, poly zone restaurant,

Fikir restaurant, and at other restaurant) in month 1(present) is equal to 0.046 + 0.22 + 0.10 + 0 = **0.370688.**

- The proportion of customers that are going to be served in Fikir restaurant after two months transition or at the third month (from those who are served at student restaurant, poly zone restaurant, Fikir restaurant, and at other restaurant) in month 1 (present) is equal to 0.052 + 0.1222 + 0.08734 + 0 = **0.261537.**
- The proportion of customers that are going to be served in other restaurant after two months transition or at the third month (from those who are served at student restaurant, poly zone restaurant, Fikir restaurant, and at other restaurant) in month 1(present) is equal to 0.012668 + 0.1079 + 0.11266 + 0.05 = **0.283274.**

Hence, in a similar way it is possible to determine the proportion of customers that are going to be served at the four restaurants for month 4, 5, 6, 7, 8, 9, and so on.

In general, according to the above computation results, the market share for student restaurant, Poly zone restaurant, Fikir restaurant and other restaurants at the end of month 3 would be 8.45%, 37%, 26.5 % and 28.32% respectively. From the mathematical model, one observe that by month 3, the market share of student restaurant decreased from 20% to 8.45%, the market share of Poly zone restaurant decreased from 45% to 37%, the market share of Fikir restaurant slightly decreased from 30% to 26.5%% but that of other restaurants increases from 5% to 11.40%. Again, if the market shares computation is repeated for subsequent future months would continue until at steady state (or equilibrium) condition when the relative market shares would no more change from year to year. Therefore, at equilibrium or steady state condition, the market share of the four restaurants in poly campus at the long-run are 0%, 0%, 0%, and 100% respectively. Note that the steady state, state probability (market share) for all restaurants depend solely on the nature of the transition probabilities and not on initial state probabilities (market share) for each of the four restaurants. Clearly, the three restaurants are dominated by other restaurants. Other restaurants are still leading the market share.

4. Conclusion

In this study, a survey is conducted on the market share of four restaurants around Bahirdar technology institute of the Bahirdar University. The result is transformed into a Markov Matrix and the switching between the four restaurants in a long period of time is observed. Due to the fact that the matrix that is formed is a systematic matrix, it is possible to reach the steady state market share or the optimum marketing mix can be reached and maintained only if no provider takes action which alters the matrix of transition probabilities. However, the optimum marketing mix or the equilibrium market shares of the providers so far are students' restaurant 0%, poly zone restaurant 0%, Fikir restaurant 0% and other restaurants 100 %. According to the steady state condition described above, it is observed that the customers' preference is intensified on the fourth restaurants named as "other restaurants". So the retention and loss of customers from each restaurant affect key business variables such as profitability and market share of the restaurants. At the steady state condition, the three restaurants loss all their profits and other restaurants become too much profitable.

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Biography of the Corresponding Author

My Name is **Amare Matebu Kassa.** I received my B.Sc. degree in Textile Engineering from Bahirdar University, Institute of Technology in 2002, the M.Sc. degree in Industrial Engineering from Addis Ababa University, Addis Ababa Institute of Technology in 2006, and the Ph.D. degree in Industrial Engineering from Addis Ababa University, Ethiopia and Politechnico di Torino, Italy, in 2011. I was a Technical Assistant, Assistant lecturer, and Lecturer in the Department of Textile Engineering, Bahirdar University, in 1995, 2002 and 2006 respectively. Since February 2012, I am an Assistant Professor in the Department of Industrial Engineering, Faculty of Mechanical and Industrial Engineering, Bahirdar Institute of Technology, Bahirdar University. My research interests include: quality engineering and management, productivity engineering and management, Maintenance engineering and management, Ergonomics, Production planning and control. Currently, I am also work as a lead researcher in Industrialization Development Policy, FDRE Policy Study and Research Center.