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Application of Krylov Subspace Improved Trajectory Piecewise-Linear Method in Reservoir Simulation

Cao Jing

（*School of Information and Mathematics, Yangtze University, Jingzhou,434023, China*）

Abstract: Improving reservoir simulation computing speed is the urgent problem to be solved under the existing calculation conditions. At present, the trajectory piecewise-linear (TPWL) reduced order method can be applied to the nonlinear reservoir simulation system. But its disadvantage is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In this paper, we improve the TPWL method from the choice of linear expansion points and the weighting function, and then reducing order of each linear model using Krylov subspace. It is called Krylov-ITPWL. We apply the method for a two-phase (oil-water) reservoir model which is solved by full implicit. The example demonstrates that which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.

Keywords: reservoir simulation; model order reduction; trajectory piecewise-linear; Krylov subspace

Traditional numerical reservoir simulators solve a set of governing partial differential equations, and then solve a set of nonlinear algebraic equations by using iteration. Because the reservoir models of real fields consist of hundreds of thousands or millions of grid blocks, these numerical solutions can be quite time consuming. Therefore, in the case of ensuring the sufficient accuracy of numerical solution, how to greatly accelerate the reservoir simulation speed is the urgent problem to be solved. Model order reduction (MOR) [1-2] techniques have shown promise in alleviating computational demands. For now, TPWL [3-6] reduced order method is be widely used in the nonlinear system. The nonlinear system can be represented as a weighted combined piecewise linear system. The TPWL method is more efficient for the model reduction of nonlinear systems, but the disadvantage of this method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In this paper, we improve the TPWL method from the choice of linear expansion points and the weighting function, and then reducing order of each linear model using Krylov subspace method. This method is called Krylov-ITPWL. We apply the method for a two-phase (oil-water) reservoir model which is solved by full implicit, which can greatly reduce the dimension of reservoir model, so as to improve the operation speed.

1. Reservoir Model

A two dimensional oil-water two phase reservoir model is used. It is assumed that oil and water do not exchange material, the process is isothermal, the fluid is compressible, and the mass conservation equation and Darcy's law can be used to obtain [7]:

$$-\nabla \bullet \left[\frac{k_{ri}\rho_i}{\mu_i} \mathbf{K}(\nabla p_i - \rho_i g \nabla d) \right] + \frac{\partial(\phi S_i \rho_i)}{\partial t} - \rho_i q_i''' = 0 \qquad (1)$$

Where $\mathbf{K}$ is permeability tensor; μ is fluid viscosity; k_r is relative permeability; p is pressure; g is gravity acceleration; d is depth; fluid density; ϕ is porosity; S is fluid saturation; t is time; q''' is a source term expressed as flow rate per unit volume; superscript $i \in \{o, w\}$ is respectively oil phase and water phase. In the equation (1), there are four unknown quantities, p_w and S_o are eliminated by using the auxiliary equation (2) and (3), so that only the state variables p_o, S_w are included in the equation,

$$S_o + S_w = 1 \tag{2}$$

$$p_o - p_w = p_c(S_w) \tag{3}$$

Where $p_c(S_w)$ is oil-water two-phase capillary pressure.

We consider the relatively simple cases and ignore gravity and capillary force. Format to discrete in space by using five point block centered finite difference, we may have the nonlinear first-order differential equation (4), see the specific derivation of literature [8]:

$$\underbrace{\begin{bmatrix} \mathbf{V}_{wp} & \mathbf{V}_{ws} \\ \mathbf{V}_{op} & \mathbf{V}_{os} \end{bmatrix}}_{\mathbf{V}} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{s}} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{T}_{w} & \mathbf{0} \\ \mathbf{T}_{o} & \mathbf{0} \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{F}_{w}(\mathbf{s}) \\ \mathbf{F}_{o}(\mathbf{s}) \end{bmatrix}}_{\mathbf{F}} \mathbf{q}_{well,t} \tag{4}$$

Where: vector $\mathbf{p}$ and $\mathbf{s}$ is grid center oil pressure p_o and water saturation S_w respectively; $\dot{\mathbf{p}}$ and $\dot{\mathbf{s}}$ is the time t derivative of vector $\mathbf{p}$ and $\mathbf{s}$ respectively; $\mathbf{V}$ is the cumulative matrix; $\mathbf{T}$ is transmission matrix; $\mathbf{F}$ is divided flow matrix; Vector $\mathbf{q}_{well,t}$ is the total flow of oil-water well.

Define the state vector $\mathbf{x}$, input vector $\mathbf{u}$ and output vector $\mathbf{y}$

$$\mathbf{x} \triangleq \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} \qquad \mathbf{u} \triangleq \begin{bmatrix} \breve{\mathbf{q}}_{well,t} \\ \breve{\mathbf{p}}_{well} \end{bmatrix} \qquad \mathbf{y} \triangleq \begin{bmatrix} \bar{\mathbf{p}}_{well} \\ \bar{\mathbf{q}}_{well,w} \\ \bar{\mathbf{q}}_{well,o} \end{bmatrix} \tag{5,6,7}$$

Where vector $\breve{\mathbf{q}}_{well,t}$ and $\breve{\mathbf{p}}_{well}$ represent the well of the constant flow and the bottom hole pressure respectively; The vector $\bar{\mathbf{p}}_{well}$ indicates the output bottom hole flow pressure of the constant flow well; Vector $\bar{\mathbf{q}}_{well,o}$ and $\bar{\mathbf{q}}_{well,w}$ indicate the output oil and water flow of the constant bottom hole pressure respectively.

The equation (4) can be written as the form of state space equation [8]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} \tag{8}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{u}) = \mathbf{C}(\mathbf{x})\mathbf{x} + \mathbf{D}(\mathbf{x})\mathbf{u} \tag{9}$$

In the control system, $\mathbf{A}$ is called the system matrix, $\mathbf{B}$ is called the input matrix, $\mathbf{C}$ is called the output matrix, $\mathbf{D}$ is called the direct transfer matrix. Because the elements of the matrix $\mathbf{V}$, $\mathbf{T}$, $\mathbf{F}$, $\mathbf{J}$ are function of the state variables, the system is a nonlinear system.

2. Krylov-ITPWL Reduced Order Method

By using the TPWL method, a set of linearized points is obtained by using a kind of linear expansion point selection algorithm: $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}$.Near the linearization points, a set of linear models are obtained by the linear expansion of the nonlinear term $\mathbf{f(x)} = \mathbf{A(x)x}$:

$$\dot{\mathbf{x}} = \mathbf{G_i x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u},\quad i = 0, 1, \cdots, (s-1) \tag{10}$$

Where: $\mathbf{G_i}$ is Jacobian matrix of $\mathbf{f(x)}$ at $\hat{\mathbf{x}}_\mathbf{i}$, $\mathbf{B}_i = \mathbf{B}(\hat{\mathbf{x}}_i)$.

By using weighted function, the approximate reduction system of the nonlinear system (8) is obtained by weighted summation of the formula (10)

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u}),\qquad i = 0, 1, \cdots, (s-1) \tag{11}$$

The implicit Euler discretization is used for each linear model

$$\frac{\mathbf{x}_{k+1} - \mathbf{x}_k}{\Delta t} = \mathbf{G_i}\mathbf{x}_{k+1} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u}_k$$

Further finishing:

$$(\mathbf{I} - \Delta t\mathbf{G_i})\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta t(\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \Delta t\mathbf{B}_i\mathbf{u}_k$$

Using Krylov subspace method to generate reduced order matrix

① set $\mathbf{A}_i = \mathbf{I} - \Delta t\mathbf{G_i}$, $\mathbf{b}_i = \Delta t(\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \Delta t\mathbf{B}_i\mathbf{u}_k$, $i = 0, 1, \cdots, (s-1)$

For each pair $\mathbf{A}_i$, $\mathbf{b}_i$, using Arnoldi method to generate Krylov base $\mathbf{V} = [\mathbf{V}_1\ \mathbf{V}_2 \cdots \mathbf{V}_s]$;

② Orthogonalize the columns of $\mathbf{V}$ using the SVD algorithm and construct a new reduced order basis matrix $\mathbf{\Phi}_r = [\boldsymbol{\varphi}_1\ \boldsymbol{\varphi}_2 \cdots \boldsymbol{\varphi}_r]$。

We can get the approximation of nonlinear system (8), (9) for order reduction system

$$\dot{\mathbf{z}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{z})(\mathbf{G}_{ir}\mathbf{z} + \mathbf{V}^T(\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_{ir}\mathbf{u}) \tag{12}$$

$$\mathbf{y} = \mathbf{C}_r\mathbf{z} + \mathbf{I} \tag{13}$$

The disadvantage of TPWL [6] method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In order to obtain high quality linear expansion, we improve the algorithm in the literature [6], and propose a linear maximum error control based on global expansion point selection algorithm, which is used in reservoir simulation. The specific process of the algorithm is as follows:

1) Give the maximum error control limit α and input vector $\mathbf{u}(t)$;

2) simulate the full order reservoir simulator, save the output state vectors $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_M\}$;

3) The initial state $\mathbf{x}_0$ is taken as the first linear expansion point $\hat{\mathbf{x}}_0$, and set $i = 1$;

4) Using the TPWL method to establish a temporary model

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i \mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i \hat{\mathbf{x}}_i)) \tag{14}$$

5) The model (14) is simulated and the state vector $\{\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \cdots, \tilde{\mathbf{x}}_M\}$ is obtained;

6) $\{\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \cdots, \tilde{\mathbf{x}}_M\}$ and $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_M\}$ will be compared to find the maximum

error state $\tilde{\mathbf{x}}_k$, and record the maximum error $\eta_{\max}$ and k ;

7) If $\eta_{\max} > \alpha$, so select the first $i+1$ linearization point $\hat{\mathbf{x}}_i = \tilde{\mathbf{x}}_k$, and set $i = i+1$, then turn to 4);

If $\eta_{\max} < \alpha$, so the loop ends and the linearization point $\{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{i-1}\}$ are returned.

Compared with other methods, this method has higher quality, and the reduced order model has smaller dimension, higher accuracy and better scalability [9].

In order to obtain high precision, we improve weight function in the literature [6], as follows:

$$\omega_i(\mathbf{z}) = \left[\frac{d_{\min}}{d_i(\mathbf{z})} e^{-\frac{d_i(\mathbf{z}) - d_{\min}}{D_{\min}}} \right]^p$$

Where： $d_i(\mathbf{z}) = |\mathbf{z} - \hat{\mathbf{z}}_i|_2^2$ ， $d_{\min} = \min(d_i(\mathbf{z}))$ ， $i = 0, 1, \cdots, (s-1)$. $D_{\min}$ is the minimum distance between linearized points $\{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}\}$ .Parameter p is between 1 and 2. At last, all weight functions are standardized and satisfied $\sum_{i=0}^{s-1} \omega_i(\mathbf{z}) = 1$ 。

3. Example Verification

A two-dimensional oil-water two phase anisotropic reservoir is described. Its grid is divided into 21 * 21, and the distribution of permeability and porosity is shown in Figure 1, 2. The related parameters of reservoir model: thickness h=2m, length and width of grid $\Delta x = \Delta y$ =33m，the viscosity of the crude oil μ_o =5mPa・s, formation water viscosity μ_w =1mPa・s，comprehensive compression coefficient c_t =3.0×10^{-3}MPa^{-1}, the original formation pressure p_i =30MPa, borehole radius r_{well} =0.114m, the end point relative permeability of oil phase k_{ro}^0 =0.9, the end point relative permeability of water phase k_{rw}^0 =0.6, oil phase Corey index n_o =2.0, water phase Corey index n_w =2.0, residual oil saturation S_{or} =0.2, irreducible water saturation S_{wc} =0.2. We use anti five point method well pattern to produce. Center has a water injection well, and four corners have four production wells. We ignore gravity and capillary force.

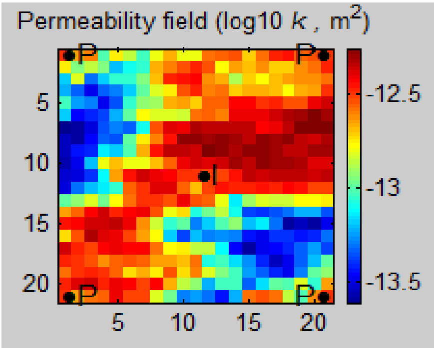

Fig.1 Permeability distribution of reservoir model

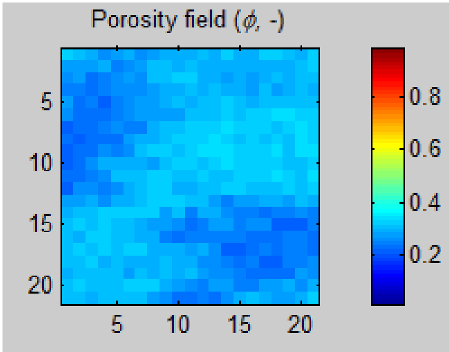

Fig.2 Porosity distribution of reservoir model

We have applied Krylov-TPWL and Krylov-ITPWL method respectively to this reservoir. To extract the information needed to reproduce the behavior of system, a full run (referred to as the training simulation) is performed. In the process of training simulation, the BHP of injection well is 35MPa, the BHPs of four production wells are 26MPa. The maximum time step allowed is 20 days. We simulate 1200 days and a total of 62 snapshots for the oil pressure and water saturation states, Jacobian matrices are recorded. For Krylov-TPWL method, we select 10 linearization points. By using the Krylov-ITPWL method, we obtain 14 linearization points.

We next consider two different scenarios to evaluate the predictive capability of Krylov-TPWL and Krylov-ITPWL reduced order model（ROM）.

(1) Schedule 1

We change the bottom-hole pressure of the four production wells, and they are set to 24MPa. The difference is smaller compared with the bottom-hole pressure of training simulation. The injection well BHP is the same as in the training simulation.

Figures 3 through 5 show the oil and water flow rates for production wells, and water injection rates for the injection well using Krylov-TPWL method. Figures 6 through 8 show the oil and water flow rates for production wells, and water injection rates for the injection well using Krylov-ITPWL method. The results of Krylov -TPWL and Krylov-ITPWL methods demonstrate close agreement with the reference simulation, but the Krylov-ITPWL method is more accurate compared with Krylov -TPWL method.

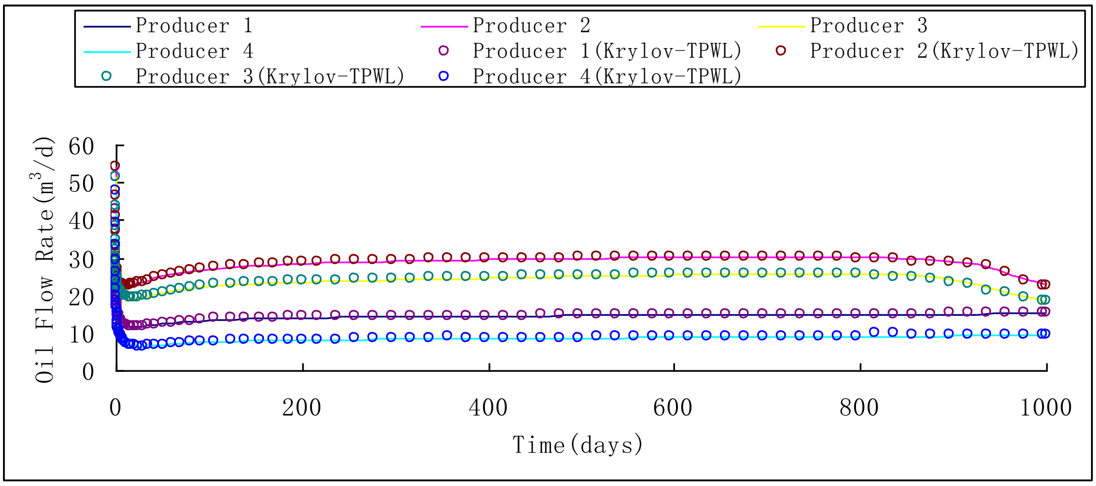

Figure 3 Oil flow rates of four production wells for Krylov-TPWL (schedule 1)

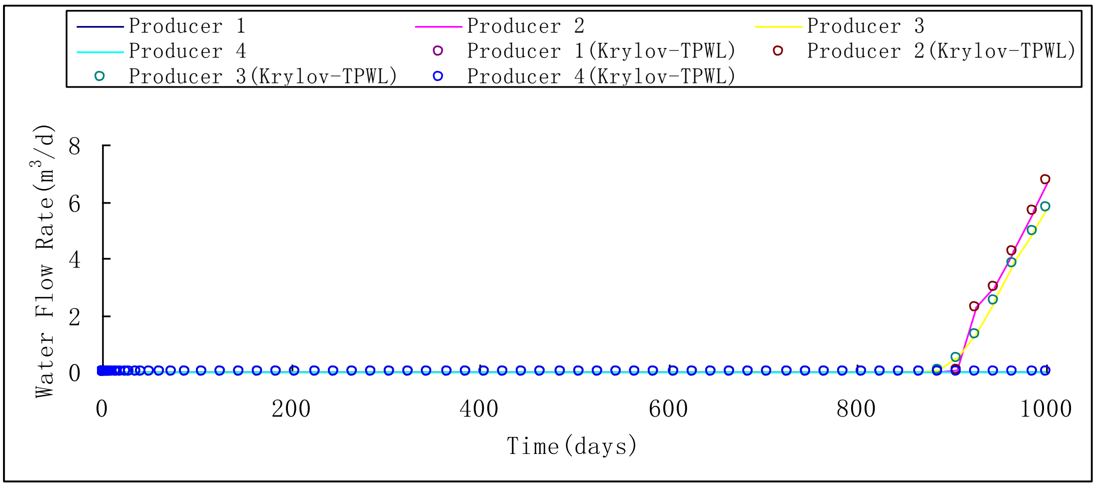

Figure 4 Water flow rates of four production wells for Krylov-TPWL (schedule 1)

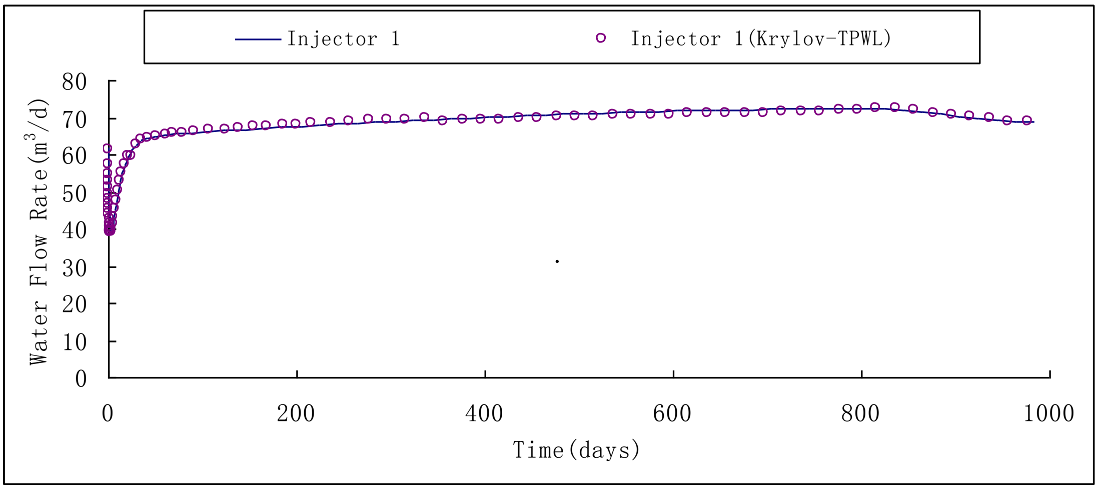

Figure 5 Water flow rate of injection well for Krylov-TPWL (schedule 1)

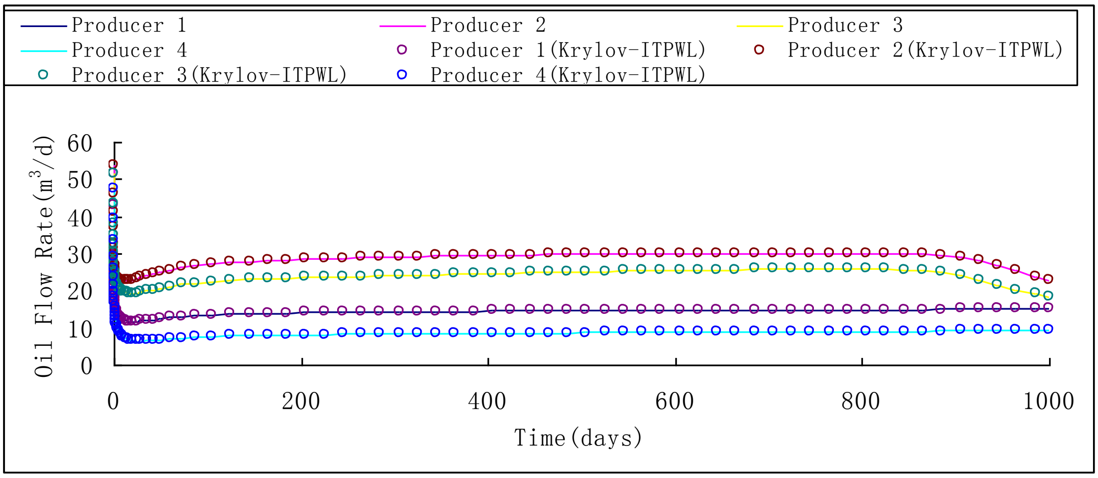

Figure 6 Oil flow rates of four production wells for Krylov-ITPWL (schedule 1)

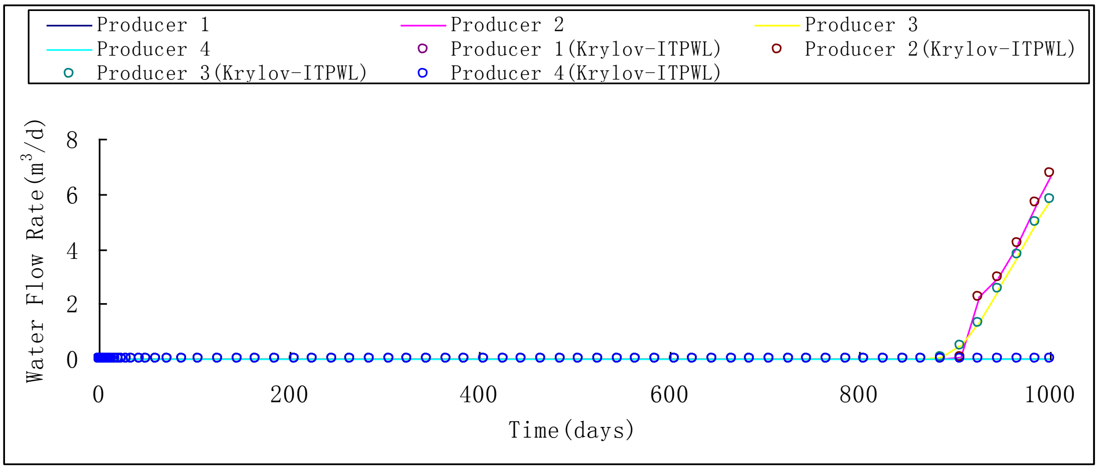

Figure 7 Water flow rates of four production wells for Krylov-ITPWL (schedule 1)

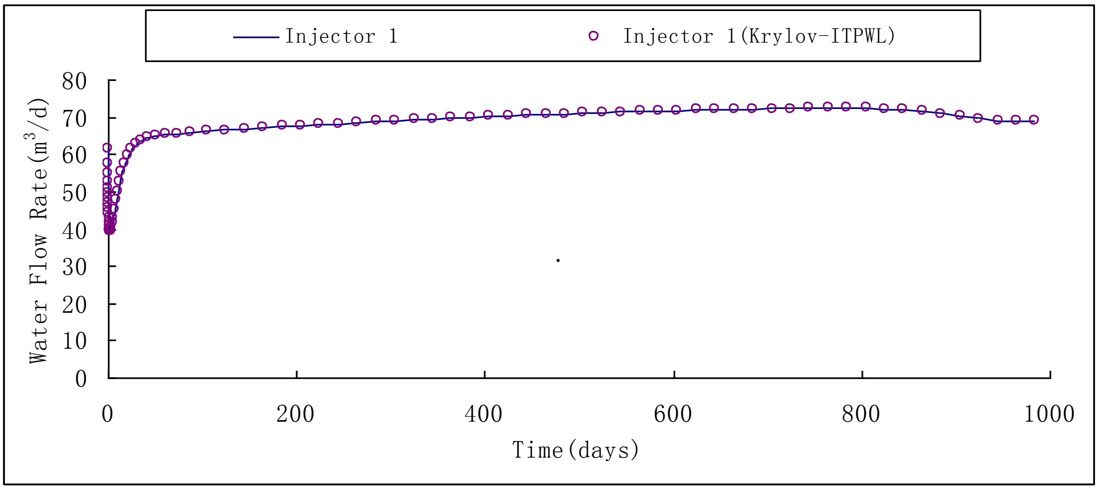

Figure 8 Water flow rate of injection well for Krylov-ITPWL (schedule 1)

The simulation times for the full-order reservoir simulation, the Krylov-TPWL reduced-order reservoir simulation, and the Krylov-ITPWL reduced-order reservoir simulation are given in table 1. The ROM with improved TPWL is able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model.

Table 1 Comparison of simulation time (schedule 1)

	full-order	Krylov-TPWL	Krylov-ITPWL
Time	95.87s	18.78s	19.82s

(2) Schedule 2

For the schedule II , four production well BHPs are set to 22MPa. The difference is larger compared with the bottom hole pressure of training simulation. The specification for the injection well is the same as in the previous case.

Figures 9 through 11 show the oil and water flow rates for production wells, and water injection rates for the injection well using Krylov-TPWL method. Figures 12 through 14 show the oil and water flow rates for production wells, and water injection rates for the injection well using Krylov-ITPWL method. The results demonstrate that when the difference of production well BHPs is larger compared with the bottom hole pressure of training simulation, the accuracy of Krylov-TPWL method becomes very poor, while the accuracy of Krylov-ITPWL method is still high.

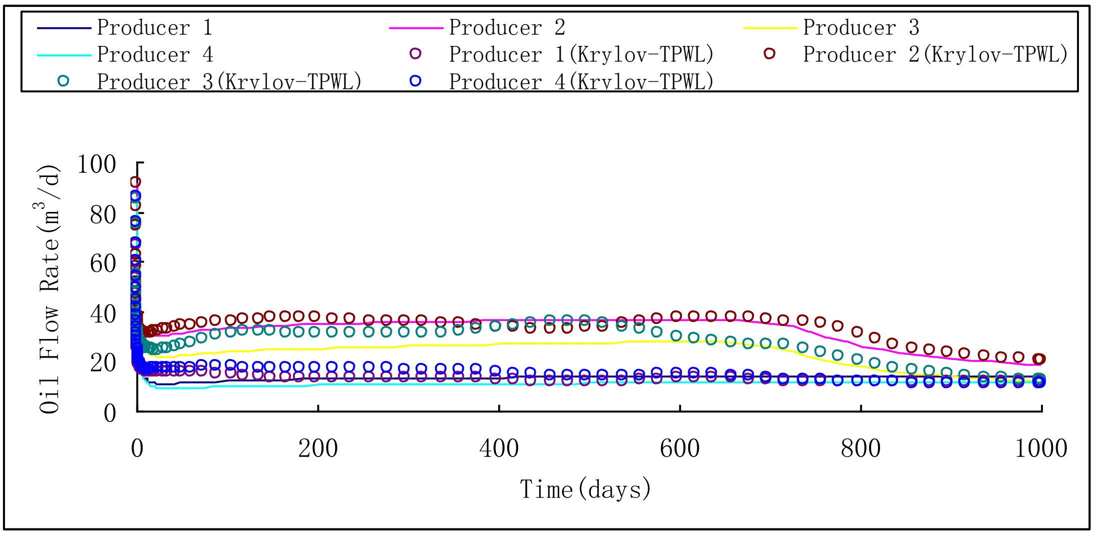

Figure 9 Oil flow rates of four production wells for Krylov-TPWL (schedule 2)

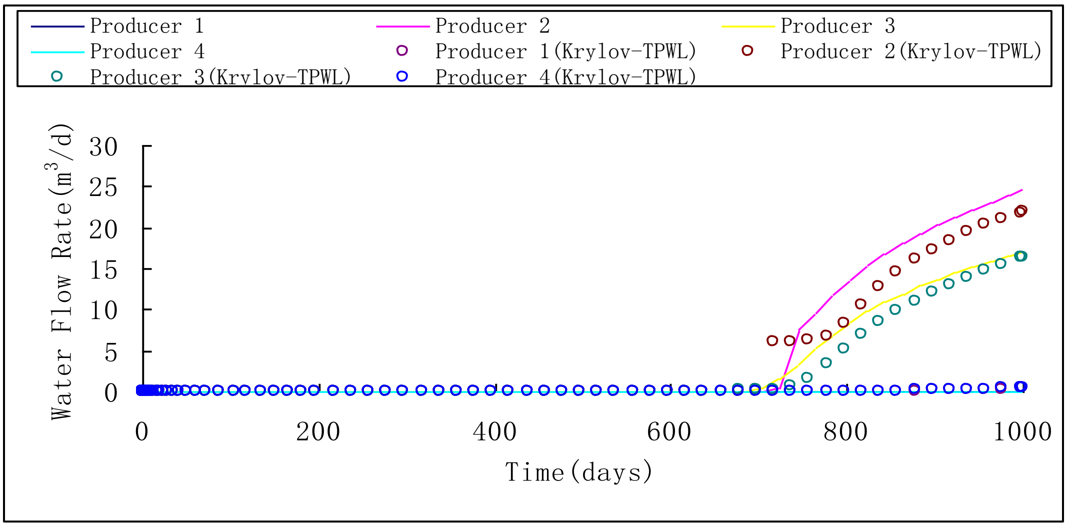

Figure 10 Water flow rates of four production wells for Krylov-TPWL (schedule 2)

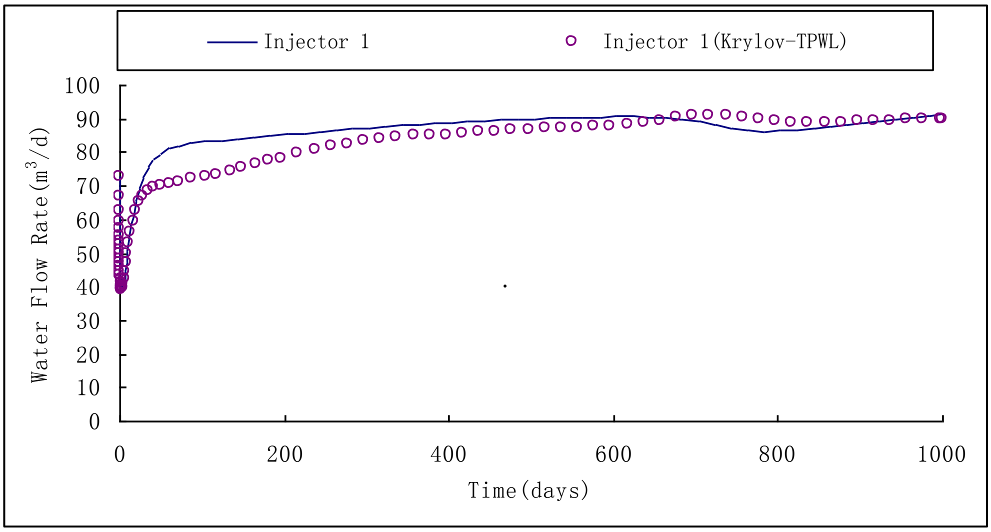

Figure 11 Water flow rate of injection well for Krylov-TPWL (schedule 2)

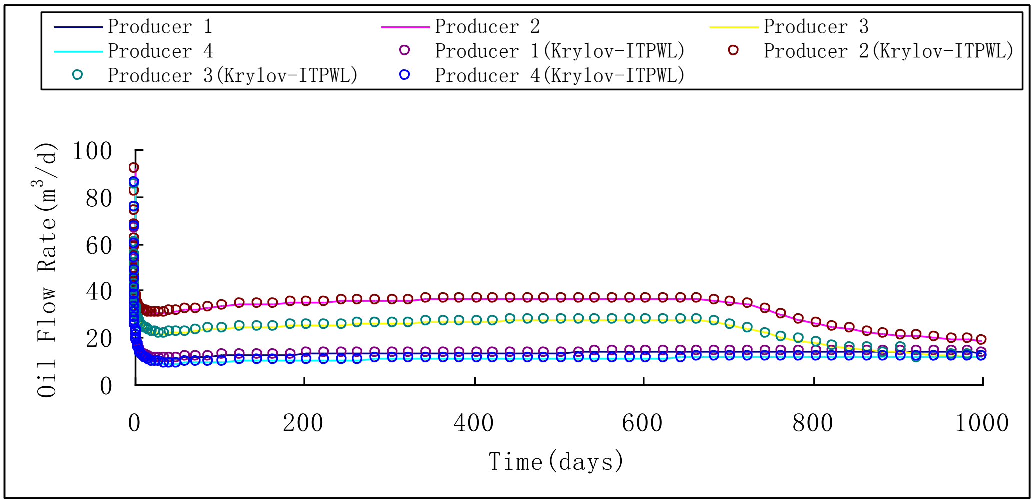

Figure 12 Oil flow rates of four production wells for Krylov-ITPWL (schedule 2)

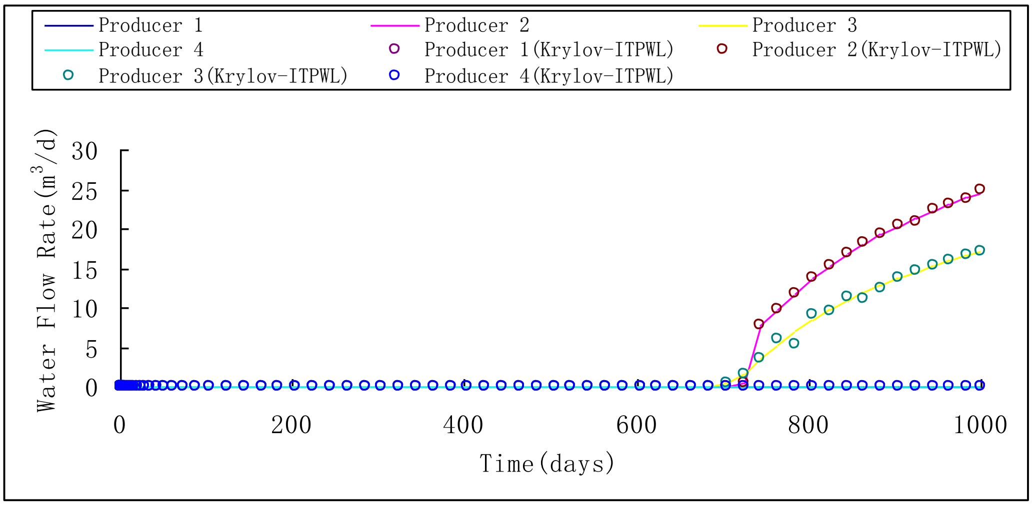

Figure 13 Water flow rates of four production wells for Krylov-ITPWL (schedule 2)

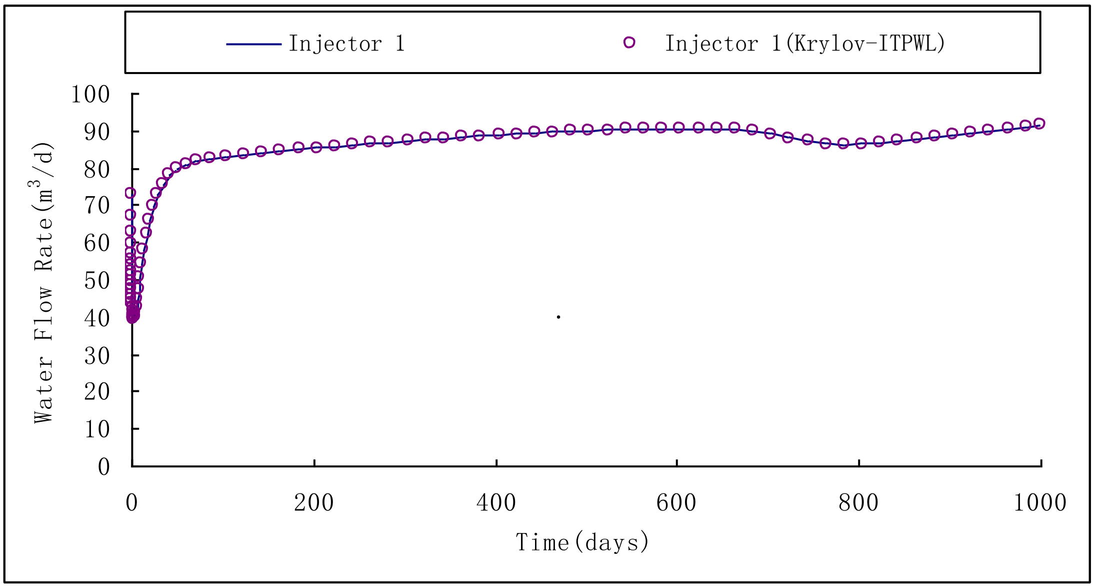

Figure 14 Water flow rate of injection well for Krylov-ITPWL (schedule 2)

For schedule II, the simulation times are given in table 2. The ROM with improved TPWL is also able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model.

Table 2 Comparison of simulation time (schedule 2)

	full-order	Krylov-TPWL	Krylov-I TPWL
Time	97.89s	17.67s	18.84s

5. Conclusion

In this work the Krylov-TPWL and Krylov-ITPWL methods are applied to a heterogeneous 2D, two-phase (oil-water) model containing 441 grid blocks and five wells. We consider two different scenarios to evaluate the predictive capability of Krylov-TPWL and Krylov-ITPWL method. The example demonstrates that if the difference of inputs of testing and training process is smaller, the results of Krylov-TPWL and Krylov-ITPWL methods ware close agreement with the full-order simulation. If the difference is larger, the accuracy of Krylov-TPWL method becomes very poor, while the accuracy of Krylov-ITPWL method is still high. And Krylov-ITPWL is able to approximately reduce the simulation time by 4 times compared with time for the full-order reservoir model. Our results show that Krylov-ITPWL outperforms Krylov-TPWL in computational accuracy.

This paper demonstrates that the use of reduced-order model based on improved TPWL appears to be a viable approach for reservoir simulation. In future work we plan to test the procedure for larger and more complicated reservoir models.

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