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Research on Improved TPWL Method for Improving Reservoir Simulation Speed

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Abstract: Improving reservoir simulation speed is the urgent problem to be solved under the existing calculation conditions. At present, although the trajectory piecewise-linear (TPWL) reduced order method can be applied to the nonlinear reservoir simulation system, the disadvantage of this method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In this paper, we improve the TPWL method from the choice of linear expansion points and the weighting function, and it is applied to reservoir simulator, which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.

Keywords: reservoir simulation; model order reduction; trajectory piecewise-linear

Reservoir simulation is an indispensable tool. It enables engineers to better understand fluid flow and to predict hydrocarbon recovery. Traditional reservoir simulators numerically solve a set of governing partial differential equations, and it entails solving a set of nonlinear algebraic equations by using iteration. Because the reservoir simulation models arising from real fields may consist of hundreds of thousands or millions of grid blocks, these numerical solutions can be quite time consuming. Therefore, in the case of ensuring the sufficient accuracy of numerical solution, how to greatly accelerate the reservoir simulation speed is the urgent problem to be solved.

Model order reduction (MOR) [1-9] techniques have shown promise in alleviating computational demands with minimal loss of accuracy. MOR is the transformation of high-dimensional models into meaningful representations of reduced dimensionality. For now, the trajectory piecewise-linear (TPWL) [10-15] reduced order method is be widely used in the nonlinear system. The nonlinear system can be represented as a weighted combined piecewise linear system. The TPWL method is more efficient for the model reduction of nonlinear systems, but the disadvantage of this method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In this paper, we improve the TPWL method from the choice of linear expansion points and the weighting function, and it is applied to reservoir simulator, which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.

1. Reservoir Model

In this paper, the mathematical model of reservoir model is transformed into the state space equation by means of space discrete in order to explain the reduction process of TPWL method. Two dimensional oil-water two phase reservoir model is used. It is assumed that oil and water do not exchange material, the process is isothermal, the fluid is compressible, and the mass conservation equation and Darcy's law can be used to obtain [16]:

$$-\nabla \bullet \left[\frac{k_{ri}\rho_i}{\mu_i} \mathbf{K}(\nabla p_i - \rho_i g \nabla d) \right] + \frac{\partial(\phi S_i \rho_i)}{\partial t} - \rho_i q_i^{'''} = 0 \quad (1)$$

Where $\mathbf{K}$ is permeability tensor; μ is fluid viscosity; k_r is relative permeability; p is pressure; g is gravity acceleration; d is depth; fluid density; ϕ is porosity; S is fluid saturation; t is time; $q^{'''}$ is a source term expressed as flow rate per unit volume; superscript $i \in \{o, w\}$ is respectively oil phase and water phase. In the equation (1), there are four unknown quantities, p_w and S_o are eliminated by using the auxiliary equation (2) and (3), so that only the state variables p_o, S_w are included in the equation,

$$S_o + S_w = 1 \quad (2)$$

$$p_o - p_w = p_c(S_w) \quad (3)$$

Where $p_c(S_w)$ is oil-water two-phase capillary pressure.

We consider the relatively simple cases and ignore gravity and capillary force. Format to discrete in space by using five point block centered finite difference, we may have the nonlinear first-order differential equation (4), see the specific derivation of literature [17]:

$$\underbrace{\begin{bmatrix} \mathbf{V}_{wp} & \mathbf{V}_{ws} \\ \mathbf{V}_{op} & \mathbf{V}_{os} \end{bmatrix}}_{\mathbf{V}} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{s}} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{T}_w & \mathbf{0} \\ \mathbf{T}_o & \mathbf{0} \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{F}_w(\mathbf{s}) \\ \mathbf{F}_o(\mathbf{s}) \end{bmatrix}}_{\mathbf{F}} \mathbf{q}_{well,t} \quad (4)$$

Where: vector $\mathbf{p}$ and $\mathbf{s}$ is grid center oil pressure p_o and water saturation S_w respectively; $\dot{\mathbf{p}}$ and $\dot{\mathbf{s}}$ is the time t derivative of vector $\mathbf{p}$ and $\mathbf{s}$ respectively; $\mathbf{V}$ is the cumulative matrix; $\mathbf{T}$ is transmission matrix; $\mathbf{F}$ is divided flow matrix; Vector $\mathbf{q}_{well,t}$ is the total flow of oil-water well.

Define the state vector $\mathbf{x}$, input vector $\mathbf{u}$ and output vector $\mathbf{y}$

$$\mathbf{x} \triangleq \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} \qquad \mathbf{u} \triangleq \begin{bmatrix} \breve{\mathbf{q}}_{well,t} \\ \breve{\mathbf{p}}_{well} \end{bmatrix} \qquad \mathbf{y} \triangleq \begin{bmatrix} \bar{\mathbf{p}}_{well} \\ \bar{\mathbf{q}}_{well,w} \\ \bar{\mathbf{q}}_{well,o} \end{bmatrix} \quad (5,6,7)$$

Where vector $\breve{\mathbf{q}}_{well,t}$ and $\breve{\mathbf{p}}_{well}$ represent the well of the constant flow and the bottom hole pressure respectively; The vector $\bar{\mathbf{p}}_{well}$ indicates the output bottom hole flow pressure of the constant flow well; Vector $\bar{\mathbf{q}}_{well,o}$ and $\bar{\mathbf{q}}_{well,w}$ indicate the output oil and water flow of the constant bottom hole pressure respectively.

The equation (4) can be written as the form of state space equation [17]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} \qquad (8)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{u}) = \mathbf{C}(\mathbf{x})\mathbf{x} + \qquad (9)$$

In the control system, $\mathbf{A}$ is called the system matrix, $\mathbf{B}$ is called the input matrix, $\mathbf{C}$ is called the output matrix, $\mathbf{D}$ is called the direct transfer matrix. Because the elements of the matrix $\mathbf{V}$, $\mathbf{T}$, $\mathbf{F}$, $\mathbf{J}$ are function of the state variables, the system is a nonlinear system.

2. TPWL Reduced Order Method

By using the TPWL [15] method, a set of linearized points is obtained by using a kind of linear expansion point selection algorithm: $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}$.Nnear the linearization points, a set of linear models are obtained by the linear expansion of the nonlinear term $\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$:

$$\dot{\mathbf{x}} = \mathbf{G}_\mathbf{i}\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G}_\mathbf{i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u}, \ i = 0, 1, \cdots, (s-1) \qquad (10)$$

Where: $\mathbf{G}_\mathbf{i}$ is Jacobian matrix of $\mathbf{f}(\mathbf{x})$ at $\hat{\mathbf{x}}_\mathbf{i}$, $\mathbf{B}_i = \mathbf{B}(\hat{\mathbf{x}}_i)$.

By using weighted function, the approximate reduction system of the nonlinear system (8) is obtained by weighted summation of the formula (10)

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G}_\mathbf{i}\hat{\mathbf{x}}_i)) \qquad (11)$$

In the literature [15], the proposed algorithm for generating the collection of linearized models may be summarized in the following steps:

1) Generate a linearized model about the initial state $\hat{\mathbf{x}}_0 = \mathbf{x}_0$, and set $i = 0$

2) Simulate the nonlinear system while $\min_{0 \le j \le i} \left\| \mathbf{x} - \mathbf{x}_j \right\| > \delta$ for some $\delta > 0$,

i.e. while the current state x is close enough to any of the previous linearization points;

3) Generate a new linearized model about $\hat{\mathbf{x}}_{i+1} = \mathbf{x}$, and set $i := i + 1$

4) If $i < s - 1$, return to step 2.

In the literature [15], the calculation of the weight function $\omega_i(\mathbf{x})$ of the current state $\mathbf{x}$ is as follows:

1) For $i = 0, 1, \cdots, (s-1)$ compute $d_i = \left\| \mathbf{x} - \hat{\mathbf{x}}_i \right\|_2$

2) Take $m = \min_{i=0,\cdots,(s-1)} d_i$

3) For $i = 0, 1, \cdots, (s-1)$ compute $\hat{\omega}_i = e^{-\beta d_i / m}$,take $\beta = 25$

4) Normalize $\hat{\omega}_{ii}$ at the evaluation point:

a) compute $S(\mathbf{x}) = \sum_{j=0}^{s-1} \hat{\omega}_j(\mathbf{x})$;

b) For $i = 0, 1, \cdots, (s-1)$, set $\omega_i(\mathbf{x}) = \hat{\omega}_i(\mathbf{x}) / S(\mathbf{x})$.

3. Improved TPWL Reduced Order Method

The disadvantage of TPWL [15] method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In order to obtain high quality linear expansion, we improve the algorithm in the literature [15], and propose a linear maximum error control based on global expansion point selection algorithm, which is used in reservoir simulation. The specific process of the algorithm is as follows:

1) Give the maximum error control limit α and input vector $\mathbf{u}(\mathrm{t})$;

2) simulate the full order reservoir simulator, save the output state vectors $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_M\}$;

3) The initial state $\mathbf{x}_0$ is taken as the first linear expansion point $\hat{\mathbf{x}}_0$, and set $i = 1$;

4) Using the TPWL method to establish a temporary model

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G}_\mathbf{i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u}) \tag{12}$$

5) The model (12) is simulated and the state vector $\{\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \cdots, \tilde{\mathbf{x}}_M\}$ is obtained;

6) $\{\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \cdots, \tilde{\mathbf{x}}_M\}$ and $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_M\}$ will be compared to find the maximum

error state $\tilde{\mathbf{x}}_k$, and record the maximum error $\eta_{\max}$ and k;

7) If $\eta_{\max} > \alpha$, so select the first $i+1$ linearization point $\hat{\mathbf{x}}_i = \tilde{\mathbf{x}}_k$, and set $i = i+1$, then turn to 4);

If $\eta_{\max} < \alpha$, so the loop ends and the linearization point $\{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{i-1}\}$ are returned.

Compared with other methods, this method has higher quality, and the reduced order model has smaller dimension, higher accuracy and better scalability [18].

In order to obtain high precision, we improve weight function in the literature [15], as follows:

$$\omega_i(\mathbf{x}) = \left[\frac{d_{\min}}{d_i(\mathbf{x})} e^{-\frac{d_i(\mathbf{x}) - d_{\min}}{D_{\min}}} \right]^p$$

Where: $d_i(\mathbf{x}) = \left|\mathbf{x} - \hat{\mathbf{x}}_i\right|_2^2$, $d_{\min} = \min(d_i(\mathbf{x}))$, $i = 0, 1, \cdots, (\mathrm{s}-1)$. $D_{\min}$ is the minimum distance between linearized points $\{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}\}$ .Parameter p is between 1 and 2. At last, all weight functions are standardized and satisfied $\sum_{i=0}^{s-1}\omega_i(\mathbf{x})=1$。

4. Example Verification

In this example, a two-dimensional oil-water two phase anisotropic reservoir is described. Its grid is divided into 21 * 21, and the distribution of permeability and porosity is shown in Figure 1, 2. The related parameters of reservoir model: thickness h=2m, length and width of grid $\Delta x=\Delta y$ =33.33m，the viscosity of the crude oil μ_o =5mPa·s, formation water viscosity μ_w =1mPa·s，comprehensive compression coefficient c_t =3.0×10^{-3}MPa^{-1}, the original formation pressure p_i =30MPa，borehole radius r_{well} =0.114m，the end point relative permeability of oil phase k_{ro}^0 =0.9，the end point relative permeability of water phase k_{rw}^0 =0.6，oil phase Corey index n_o =2.0, water phase Corey index n_w =2.0, residual oil saturation S_{or} =0.2, irreducible water saturation S_{wc} =0.2. We use anti five point method well pattern to produce. Center has a water injection well, and four corners have four production wells. We ignore gravity and capillary force.

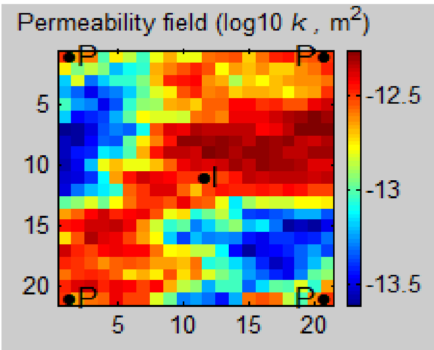

Fig.1 Permeability distribution of reservoir model

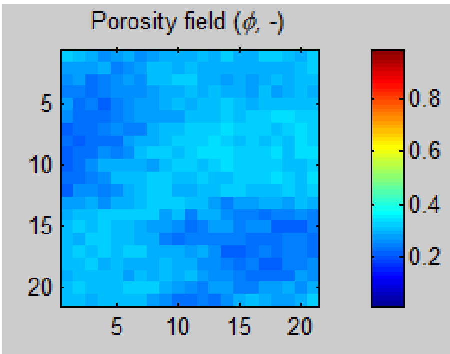

Fig.2 Porosity distribution of reservoir model

We have applied TPWL and improved TPWL method respectively to this reservoir. To extract the information needed to reproduce the behavior of system, a full run (referred to as the training simulation) is performed. In the process of training simulation, the BHP of injection well is 34MPa, the BHPs of four production wells are 26MPa. The maximum time step allowed is 20 days. We simulate 1500 days and a total of 105 snapshots for the oil pressure and water saturation states, Jacobian matrices are recorded. For TPWL method, we select 10 linearization points. By using the improved TPWL method, we obtain 13 linearization points.

We next consider two different scenarios to evaluate the predictive capability of TPWL and improved TPWL reduced order model（ROM）.

(1) Prediction Using ROM - Schedule I

We change the bottom-hole pressure of the four production wells, and they are set to 24MPa. The difference is smaller compared with the bottom-hole pressure of training simulation. The injection well BHP is the same as in the training simulation.

Figures 3 through 5 show the oil and water flow rates for production wells, and water injection rates for the injection well using TPWL method. Solid lines are used for the flow rates from the high-fidelity (reference) solution, and circles are used for TPWL solution. Figures 6 through 8 show the oil and water flow rates for production wells, and water injection rates for the injection well using improved TPWL method. Solid lines are also used for the flow rates from the high-fidelity (reference) solution, and circles are used for improved TPWL solution. The results of TPWL and improved TPWL methods demonstrate close agreement with the reference simulation, but the improved TPWL method is more accurate compared with TPWL method.

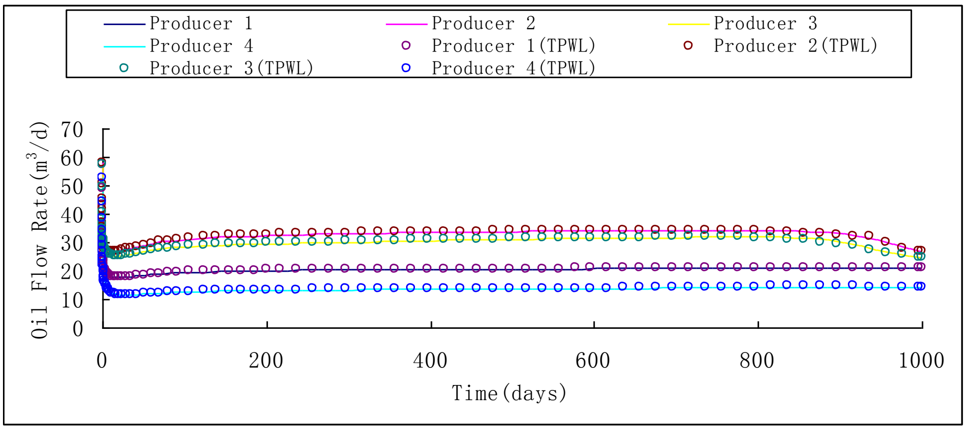

Figure 3 Oil flow rates of four production wells for TPWL (schedule I)

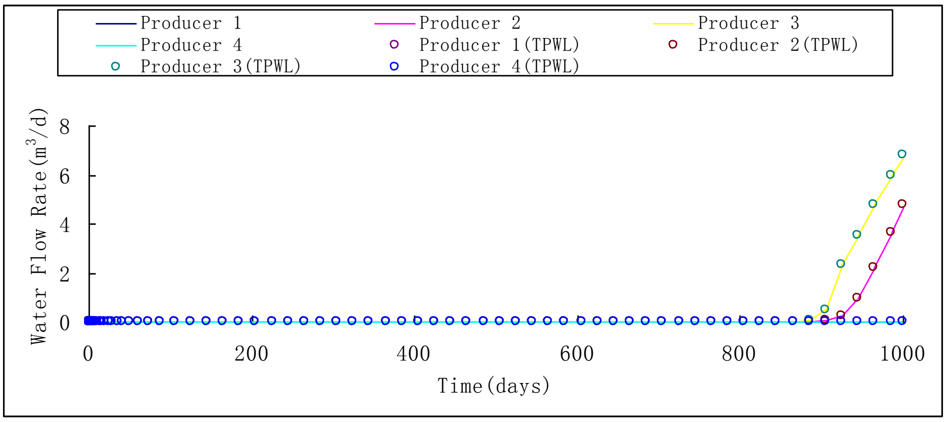

Figure 4 Water flow rates of four production wells for TPWL (schedule I)

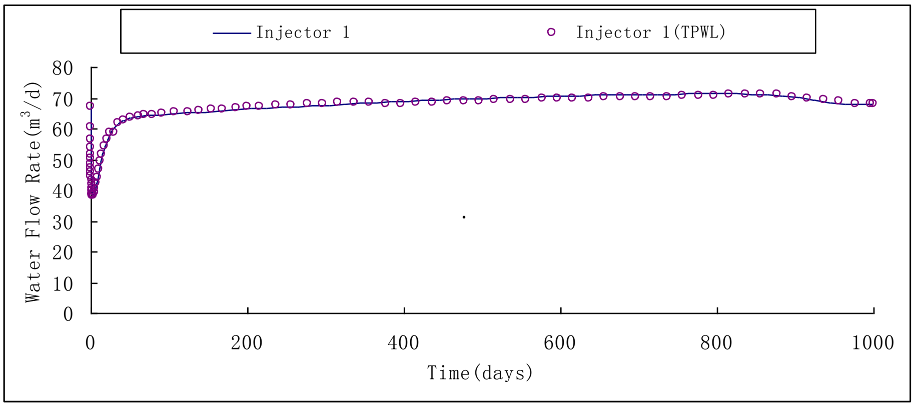

Figure 5 Water flow rate of injection well for TPWL (schedule I)

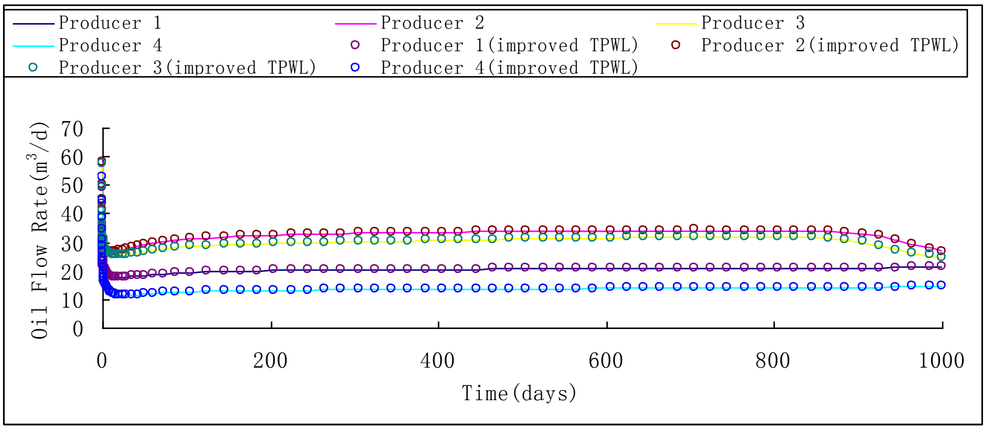

Figure 6 Oil flow rates of four production wells for improved TPWL (schedule I)

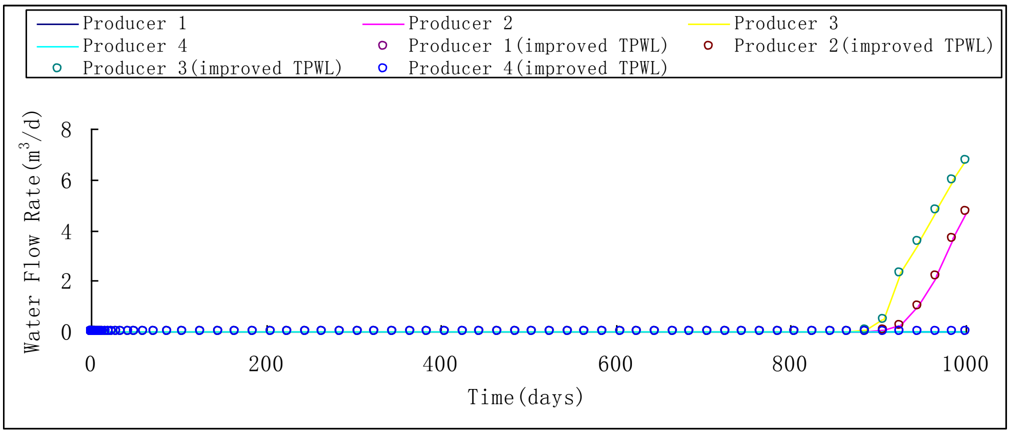

Figure 7 Water flow rates of four production wells for improved TPWL (schedule I)

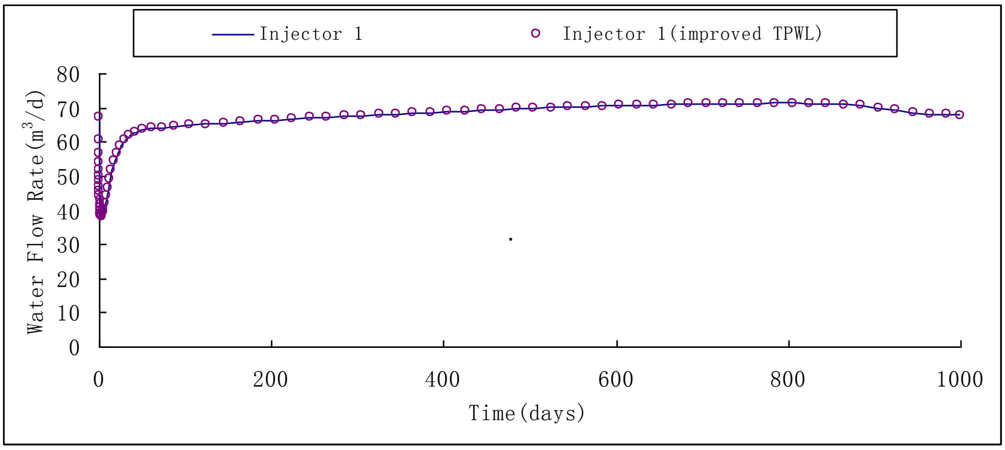

Figure 8 Water flow rate of injection well for improved TPWL (schedule I)

The simulation times for the full-order reservoir simulation, the TPWL reduced-order reservoir simulation, and the improved TPWL reduced-order reservoir simulation are given in table 1. The ROM with improved TPWL is able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model.

Table 1 Comparison of simulation time (schedule I)

	full-order	TPWL	improved TPWL

Time	94.87s	18.36s	19.53s

(2) Prediction Using ROM - Schedule II

For the schedule II , four production well BHPs are set to 22MPa. The difference is larger compared with the bottom hole pressure of training simulation. The specification for the injection well is the same as in the previous case.

Figures 9 through 11 show the oil and water flow rates for production wells, and water injection rates for the injection well using TPWL method. Figures 12 through 14 show the oil and water flow rates for production wells, and water injection rates for the injection well using improved TPWL method. The results demonstrate that when the difference of production well BHPs is larger compared with the bottom hole pressure of training simulation, the accuracy of TPWL method becomes very poor, while the accuracy of improved TPWL method is still high.

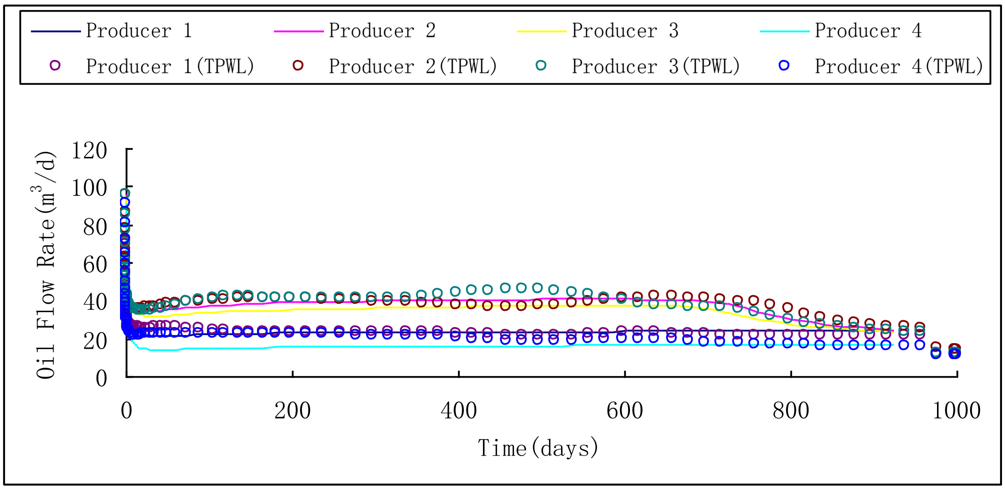

Figure 9 Oil flow rates of four production wells for TPWL (schedule II)

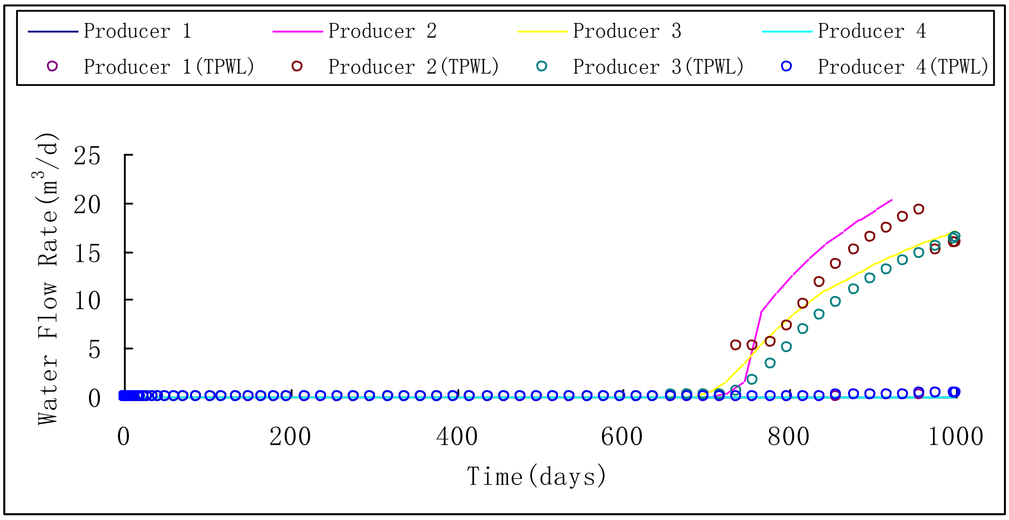

Figure 10 Water flow rates of four production wells for TPWL (schedule II)

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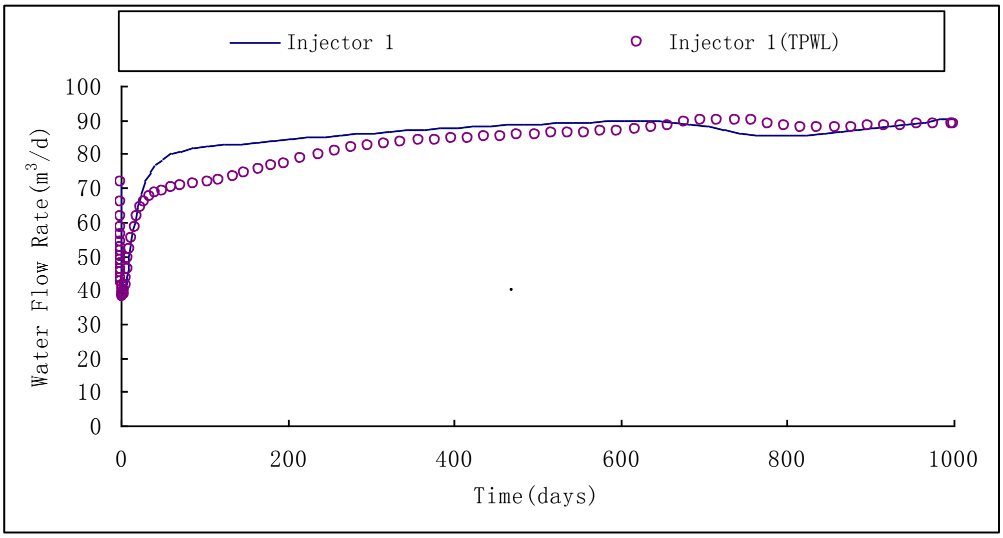

Figure 11 Water flow rate of injection well for TPWL (schedule II)

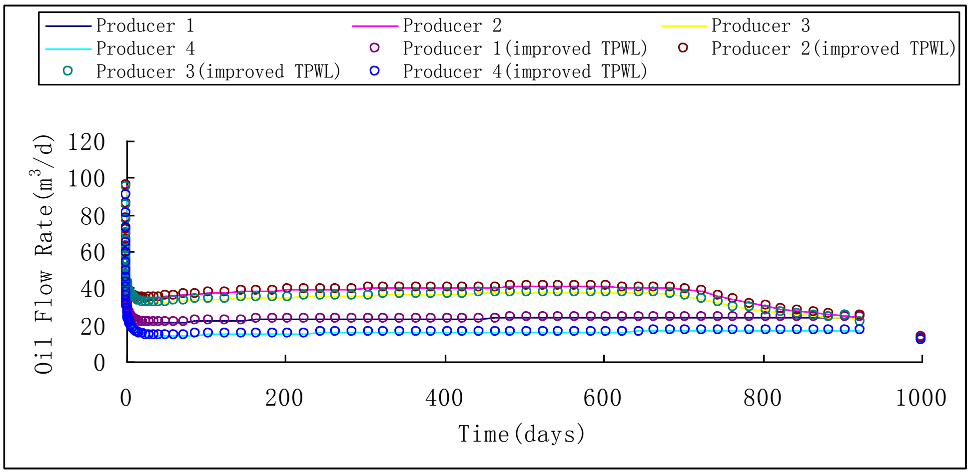

Figure 12 Oil flow rates of four production wells for improved TPWL (schedule II)

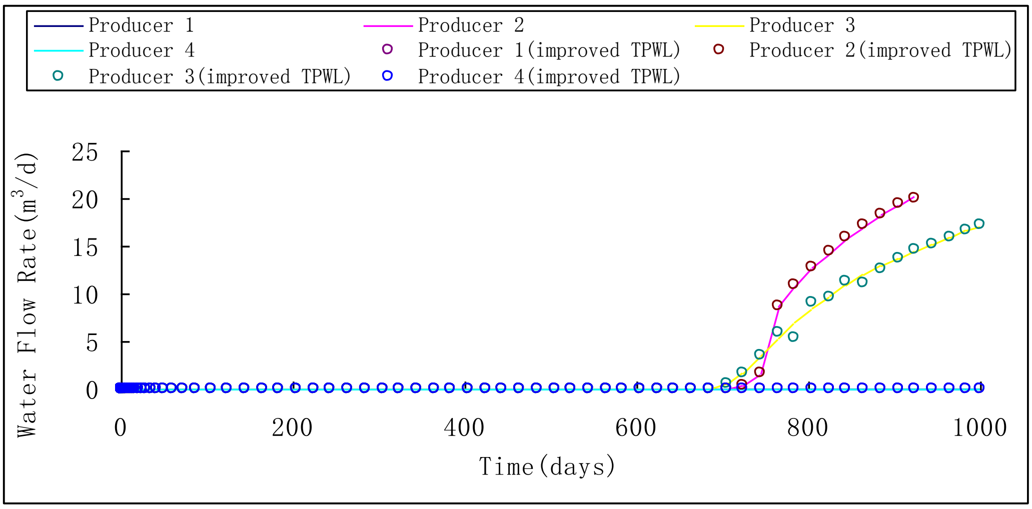

Figure 13 Water flow rates of four production wells for improved TPWL (schedule II)

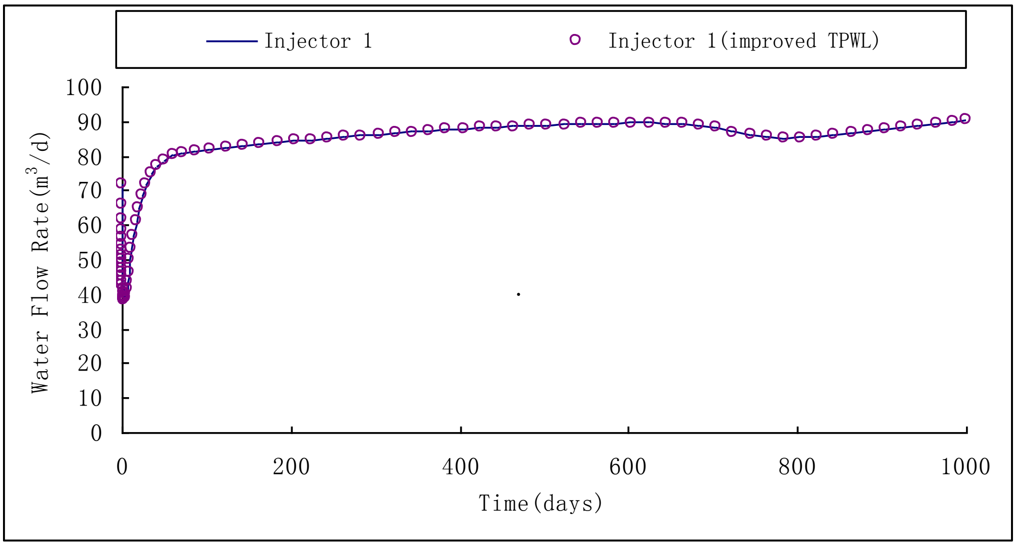

Figure 14 Water flow rate of injection well for improved TPWL (schedule II)

For schedule II, the simulation times are given in table 2. The ROM with improved TPWL is also able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model.

Table 2 Comparison of simulation time (schedule II)

	full-order	TPWL	improved TPWL
Time	97.75s	17.63s	18.59s

5. Conclusion

In this work the improved TPWL and TPWL methods are applied to a heterogeneous 2D, two-phase (oil-water) model containing 481 grid blocks and five wells. We consider two different scenarios to evaluate the predictive capability of improved TPWL and TPWL method. The example demonstrates that if the difference of inputs of testing and training process is smaller, the results of TPWL improved TPWL methods ware close agreement with the full-order simulation. If the difference is larger, the accuracy of TPWL method becomes very poor, while the accuracy of improved TPWL method is still high. And improved TPWL is able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model. Our results show that improved TPWL outperforms TPWL in computational accuracy.

This paper demonstrates that the use of reduced-order model based on improved TPWL appears to be a viable approach for reservoir simulation. In future work we plan to test the procedure for larger and more complicated reservoir models.

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