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An open repayment model with flexible parameters

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Abstract: In the paper we present some ideas for loan (debt) repayment apart from classical models. These ideas make an open repayment model which respects the debtor's financial possibilities and, at the same time, stimulates him to realize the payments regularly. In this way the presented model can be useful not only for debtor but also for creditor.
Keywords: repayment model, amortization schedule, interest rate.

1. Introduction and motivation

Nowadays a wide variety of loans on offer seem to provide an easy way to achieve our aspirations which are very often above real possibilities. But the loan is debt which have to be repaid sooner or later. In the repayment procedure many difficulties may occur over the term of annuity especially for the long ones. Rigid classical models are not fit or ready for such situations which can result with serious problems for debtor and creditor. In this paper we propose an open, flexible model where unpredictable situation is the basic assumption.

Loans and loan repayment problems are widely studied in the literature. In [2], [3] and [6] the classical loan repayment models with necessary mathematical background are given. Using structural equation modelling, in [7] the authors present econometric models for evaluating parameters of regular student loan repayment. Critical factors affecting the repayment of microcredit are examined in [4]. In contrast to the usual opinion, a large scale randomized field experiment with a typical urban microfinance institution (MFI) in [1] provides no evidence that lower frequency repayment schedules encourage irresponsible repayment behaviour among first-time borrowers receiving small loans. An objective analysis of prepayment risk, based on a study of different factors influencing bank customer behaviour and their influence on early loan repayment, is given in [5]

2. Assumptions and notation

We consider here an ordinary simple annuity which means that payments are placed at the end of each equal rent period (payment intervals) and that the interest is compounded at the same frequency as the payments are made. We shall use the following notation.

D_0 is the amount of debt (loan, principal) to be paid (the present value of annuity),
n is the number of payments in the term of annuity (the number of rent periods).

For the kth rent period, $k \in \{1, 2, ..., n\}$, we set:

i_k is the interest rate (nominal rate divided by periods per year),
R_k is the payment, P_k is the principal paid, I_k is the interest paid and
D_k is the outstanding balance (the rest of debt) after the payment.

It is well known (see [2], [3], [6]) that for the above terms the following relations hold,

$$I_k = D_{k-1} i_k, \quad R_k = P_k + I_k, \quad D_k = D_{k-1} - P_k, \quad k = 1, 2, 3, ..., n, \tag{1}$$

The relations (1) are used to create the amortization schedule. Since, in repayment procedure, debt is decreased in the period k by the principal paid P_k, we have

$$D_{k-1} = P_k + P_{k+1} + ... + P_n, \quad k = 1, 2, 3, ..., n; \quad D_n = 0,$$

and particularly $D_0 = P_1 + P_2 + ... + P_{n-1} + P_n$. Thus, the total amount of interest, which is the difference between total payments and principal, is

$$I = R_1 + R_2 + ... + R_n - D_0 = (R_1 - P_1) + (R_2 - P_2) + ... + (R_n - P_n) = I_1 + I_2 + ... + I_n.$$

3. Open repayment model

First we recall two most commonly used classical amortization models where a fixed interest rate i for each rent period is assumed.

Model of equal payments. Equal amount R has to be paid at the end of each period. The amount R and total interest I are (see [2], [3], [6])

$$R = D_0 i \frac{(1+i)^n}{(1+i)^n - 1}, \quad I = nR - D_0 = D_0 \left[\frac{ni(1+i)^n}{(1+i)^n - 1} - 1 \right]. \tag{2}$$

Model of equal principal payments. In the each payment R_k, which is placed at the end of period $k \in \{1, 2, ..., n\}$, the portion of principal P is the same. Here we have (see [2], [6])

$$P = \frac{D_0}{n}, \quad R_k = P\left[1 + (n-k+1)i\right], \ \ k = 1, 2, ..., n, \quad I = \frac{D_0 i}{2}(n+1). \tag{3}$$

Using the relations (1) and (2) or (1) and (3) the amortization schedule is created before the payments start. For the above models it is fixed for the whole term of annuity. As we mentioned earlier, over time many difficulties, concerning regular payment, may occur in the term of annuity. For that reason we propose an open model with empty amortization schedule at the beginning. The model is based on the following general rules.

- Debtor and creditor make arrangements on the parameters in the repayment procedure.
- Debtor realizes the payments as he can (or want) according to his possibilities.
- If he respects the agreement then he can be rewarded with benefits.
- If he violates the agreement then he can be punished with penalties.
- Debtor is informed of the awards and penalties.

In this way the repayment procedure is monitoring in each rent period. If the payments are made in accordance with the agreed conditions (contract) then, in the next period, debtor can be rewarded with more favourable payment terms. There are many ways to do this: decreasing the interest rate, decreasing the next payment, delay the next payment with low (or without) interest rate, increasing the number of rent periods etc. On the contrary, if the agreed conditions are violated then, in the next period, debtor can be punished with worse payment terms like: increasing the interest rate, increasing the next payment, shortening the next rent period, decreasing the number of rent periods etc.

Thus, instead of strictly determined classical models we propose an open flexible model which is adaptable to current circumstances which are almost always unpredictable. Such approach is appropriate for debtor and creditor because it simplifies the situations when difficulties arise. We shall present some features of the model in the next section.

4. Examples

We apply and explain the proposed model through the following examples.

Example 1. Suppose that 15000 pennies have to be repaid during 5 periods with interest rate of 6% using the model of equal principal payments. In the case of regular payments the interest rate will be decreased by 0.5% while, in the case of irregular payments, it will be increased by 1% and the term of annuity will be prolonged. Let us see the situation where debtor could not realize third and fifth payment.

Using (3) we have $D_0 = 15000,\ n = 5,\ P = 3000.$ We create the repayment schedule by using the relation (1).

k	i_k (%)	R_k	I_k	P_k	D_k
0	-	-	-	-	15000.00

1	6	3900.00	900.00	3000.00	12000.00
2	5.5	3660.00	660.00	3000.00	9000.00
3	5	0.00	450.00	- 450.00	9450.00
4	6	3567.00	567.00	3000.00	6450.00
5	5.5	0.00	354.75	- 354.75	6804.75
6	6.5	3442.31	442.31	3000.00	3804.75
7	6	3228.29	228.29	3000.00	804.75
8	5.5	849.01	44.26	804.75	0.00
Σ		18646.61	3646.21	15000.00	

We see that for $k = 4, 6$ the interest rate increases (penalty) while for $k = 2, 3, 5, 7, 8$ it decreases (benefit). This example can be generalized by the following relation.

$$i_k = i_R(k) - i_B(k) + i_P(k), \quad k = 1, 2, ..., n,$$

where $i_R(k)$ is regular part of interest rate in period k according to the arrangement (contract) between debtor and creditor, $i_B(k)$ is stimulating part (benefit) and $i_P(k)$ is penalty. In this example we have

$$i_1 = i_R(1) = 6\%, \quad i_B(1) = i_P(1) = 0\%,$$
$$i_R(k) = i_{k-1}, \quad i_B(k) \in \{0\%,\ 0.5\%\}, \quad i_P(k) \in \{0\%,\ 1\%\} \ \text{ for } \ k > 1.$$

Note that similar general relation could be also stated for other parameters, R_k, I_k or P_k.

Example 2. Loan of 18000 pennies is obtained at interest rate of 5%. If the loan will be repaid within 6 periods then the total amount of interest will not exceed 3200 pennies.

The only necessary condition to get the benefit is to repay the loan within 6 periods. Debtor has different possibilities to choose payment dynamics. If he pays more than 3200 pennies of total interest then he will get the difference back. Let us see some possibilities.
If he chooses the model of equal payments then, using (2) where $D_0 = 18000,\ i = 0.05,\ n = 6$, we have $R = 3546.31,\ I = 3277.89$. In this case, after the repayment procedure is completed, debtor will get 77.89 pennies back.
If he chooses the model of equal principal payments then, using (3) with the same initial values, we have $P = 3000,\ I = 3150$. In this case debtor does not need the benefit because the paid total interest is $3150 < 3200$.
Debtor will have the greatest benefit in the following model.

k	i_k (%)	R_k	I_k	P_k	D_k
0	-	-	-	-	18000.00
1	5	0.00	900.00	- 900.00	18900.00
2	5	0.00	945.00	- 945.00	19845.00
3	5	0.00	992.25	- 992.25	20837.25
4	5	0.00	1041.86	- 1041.86	21879.11
5	5	0.00	1093.96	- 1093.96	22973.07
6	5	24121.72	1148.65	22973.07	0.00
Σ		24121.72	6121.72	18000.00	

In this case debtor will get $6121.72 - 3200 = 2921.72$ pennies back. The smallest amount of total interest debtor will pay in the following model.

k	i_k (%)	R_k	I_k	P_k	D_k
0	-	-	-	-	18000.00
1	5	18900.00	900.00	18000.00	0.00
Σ		18900.00	900.00	18000.00	

Example 3. Loan of 36000 pennies is obtained at initial interest rate (for the first rent period) 1%. For each next period the interest rate is proportional to the previous principal payment: from 1% for 6000 or more up to 7% for zero or negative principle payment.

We see that debtor can repay the loan with interest rate 1% if each principal payment is at least 6000 pennies. In the periods when he is not able to realize the payment or the payment does not exceed the amount of interest, the interest rate is 7%. If the principal payment is between 0 and 6000 then the interest rate is between 7% and 1%, Thus, we have $i_1 = 1\%$ and

$$i_k(\%) = \begin{cases} 1 \text{ for } P_{k-1} > 6000, \ k > 1, \\ 1 + \dfrac{6000 - P_{k-1}}{6000 - 0} \cdot (7-1) = 7 - \dfrac{P_{k-1}}{1000} \text{ for } 0 \le P_{k-1} \le 6000, \ k > 1, \\ 7 \text{ for } P_{k-1} < 0, \ k > 1. \end{cases}$$

Suppose that the payment procedure was going in the following way.

k	i_k (%)	R_k	I_k	P_k	D_k
0	-	-	-	-	36000.00
1	1	3360.00	360.00	3000.00	33000.00
2	4	6320.00	1320.00	5000.00	28000.00
3	2	7560.00	560.00	7000.00	21000.00
4	1	4210.00	210.00	4000.00	17000.00
5	3	1510.00	510.00	1000.00	16000.00
6	6	0.00	960.00	- 960.00	16960.00
7	7	4447.20	1187.20	3260.00	13700.00
8	3.74	6212.38	512.38	5700.00	8000.00
9	1.3	4604.00	104.00	4500.00	3500.00
10	2.5	87.50	87.50	0.00	3500.00
11	7	3745.00	245.00	3500.00	0.00
Σ		42056.08	6056.08	36000.00	

In the above examples we can see that the model has high degree of flexibility. It covers and respects both, good and bad, debtor's financial possibilities.

5. Conclusions

In everyday life individual and collective (national) prosperity usually rests on debt. But, in recent unstable financial circumstances, regular debt repayment can become very difficult and problematic or even impossible. Debt rescheduling, which is necessary in such situations, could be harmful and painful for debtor. Rigid repayment models and negative attitude of creditors about debt rescheduling makes this situation even worse.

In the paper we have presented an open, flexible model for debt (loan) repayment. It rests on the assumption that unpredictable situations could occur in each rent period. General rules of the model allow debtor to choose payment dynamics according to his financial possibilities. If he keeps an agreement in principle then he can be rewarded with the better payment terms but, on the contrary, they can be made worse. He is fully informed about awards and penalties. In this way debtor is motivated to realize regular payments and, at the same time, he is not too much frightened whenever regular repayment becomes questionable. In this way the model respects debtor's financial possibilities and thus ensures flexibility and adaptability to current situations.

References

[1] Field E., Pande R., "Repayment Frequency and Default in Micro-Finance: Evidence from India"Journal of the European Economic Association 6, No. 2-3, 2008.

[2] Grofelnik J., "Osnovni modeli otplate zajma", graduate thesis, Faculty of economics and business, Zagreb 2013.

[3] Guthrie L. Gary, Lemon D. Larry, "Mathematics of Interest Rates and Finance"Pearson Education, Inc. 2010.

[4] Mamun A. Abdullah, Wahab A. Sazali, Malarvizhi C. A., Mariapun S.,"Examining the Critical Factors Affecting the Repayment of Microcredit Provided by Amanah Ikhtiar Malaysia" International Business Research 4, No. 2; 2011.

[5] Nitescu D. Costin,"Prepayment risk, impact on credit products"Theoretical and Applied Economics 19, No. 8(573), 2012.

[6] Relić B., "Gospodarska matematika" Hrvatska zajednica računovođa i financijskih djelatnika, Zagreb 2002.

[7] Shafinar I., Satwinder S., Aqilah N., Md Sahiq, "Mediation Effects of Educational Loan Repayment Model"British Journal of Economics, Finance and Management Sciences 5, No. 1, 2012.